

STUDENT SOLUTIONS MANUAL

DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS Computing and Modeling

3E

EDWARDS & PENNEY



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Computing and Modeling

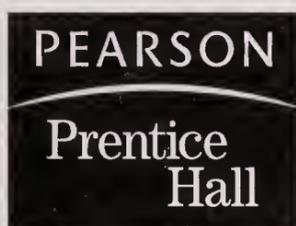
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EDWARDS & PENNEY



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PREFACE

This is a solutions manual to accompany the textbook **DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS: Computing and Modeling** (3rd edition, 2004) by C. Henry Edwards and David E. Penney. We include solutions to most of the odd-numbered problems in the text.

Our goal is to support learning of the subject of elementary differential equations in every way that we can. We therefore invite comments and suggested improvements for future printings of this manual, as well as advice regarding features that might be added to increase its usefulness in subsequent editions. Additional supplementary material can be found at our textbook Web site listed below.

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CHAPTER 1

FIRST-ORDER DIFFERENTIAL EQUATIONS

SECTION 1.1

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING

The main purpose of Section 1.1 is simply to introduce the basic notation and terminology of differential equations, and to show the student what is meant by a solution of a differential equation. Also, the use of differential equations in the mathematical modeling of real-world phenomena is outlined.

Problems 1–12 are routine verifications by direct substitution of the suggested solutions into the given differential equations. We include here just some typical examples of such verifications.

3. If $y_1 = \cos 2x$ and $y_2 = \sin 2x$, then $y_1' = -2 \sin 2x$ and $y_2' = 2 \cos 2x$ so

$$y_1'' = -4 \cos 2x = -4 y_1 \quad \text{and} \quad y_2'' = -4 \sin 2x = -4 y_2.$$

$$\text{Thus } y_1'' + 4 y_1 = 0 \text{ and } y_2'' + 4 y_2 = 0.$$

5. If $y = e^x - e^{-x}$, then $y' = e^x + e^{-x}$ so $y' - y = (e^x + e^{-x}) - (e^x - e^{-x}) = 2e^{-x}$. Thus $y' = y + 2e^{-x}$.

11. If $y = y_1 = x^{-2}$ then $y' = -2x^{-3}$ and $y'' = 6x^{-4}$, so

$$x^2 y'' + 5x y' + 4y = x^2 (6x^{-4}) + 5x (-2x^{-3}) + 4(x^{-2}) = 0.$$

If $y = y_2 = x^{-2} \ln x$ then $y' = x^{-3} - 2x^{-3} \ln x$ and $y'' = -5x^{-4} + 6x^{-4} \ln x$, so

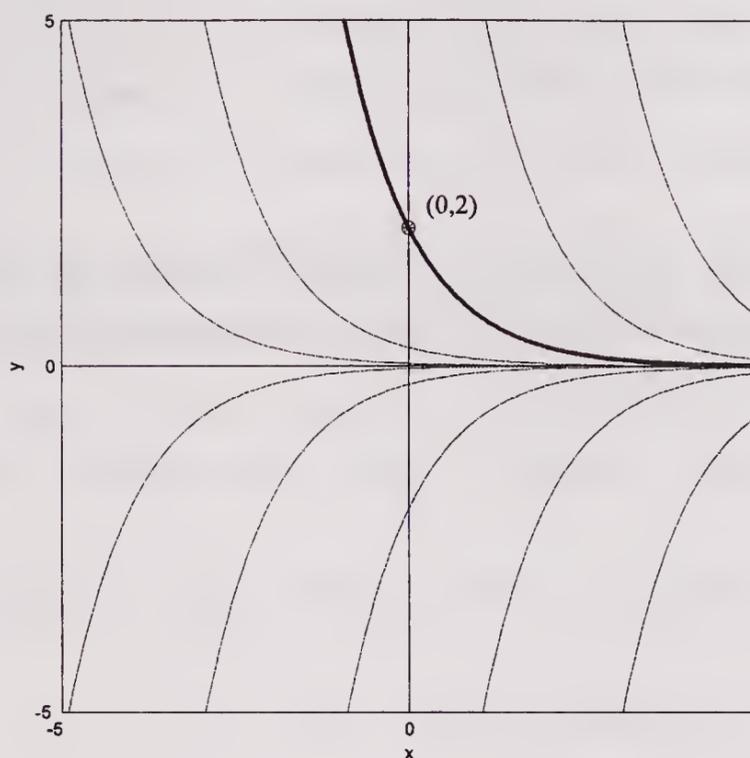
$$\begin{aligned} x^2 y'' + 5x y' + 4y &= x^2 (-5x^{-4} + 6x^{-4} \ln x) + 5x (x^{-3} - 2x^{-3} \ln x) + 4(x^{-2} \ln x) \\ &= (-5x^{-2} + 5x^{-2}) + (6x^{-2} - 10x^{-2} + 4x^{-2}) \ln x = 0. \end{aligned}$$

13. Substitution of $y = e^{rx}$ into $3y' = 2y$ gives the equation $3r e^{rx} = 2e^{rx}$ that simplifies to $3r = 2$. Thus $r = 2/3$.

15. Substitution of $y = e^{rx}$ into $y'' + y' - 2y = 0$ gives the equation $r^2 e^{rx} + r e^{rx} - 2e^{rx} = 0$ that simplifies to $r^2 + r - 2 = (r+2)(r-1) = 0$. Thus $r = -2$ or $r = 1$.

The verifications of the suggested solutions in Problems 17–26 are similar to those in Problems 1–12. We illustrate the determination of the value of C only in some typical cases. However, we illustrate typical solution curves for each of these problems.

17. $C = 2$



19. If $y(x) = C e^x - 1$ then $y(0) = 5$ gives $C - 1 = 5$, so $C = 6$.

