

# STUDENT SOLUTIONS MANUAL

---

# DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS Computing and Modeling 3E

---

EDWARDS  
& PENNEY



Digitized by the Internet Archive  
in 2018 with funding from  
Kahle/Austin Foundation

<https://archive.org/details/studentsolutions0000edwa>

# STUDENT SOLUTIONS MANUAL

---

# DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS Computing and Modeling 3E

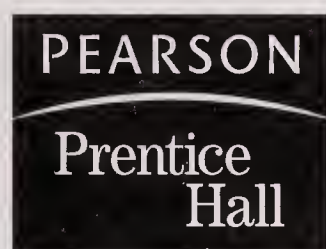
---

EDWARDS  
& PENNEY



Upper Saddle River, NJ 07458

Acquisitions Editor: George Lobell  
Supplement Editor: Jennifer Brady  
Assistant Managing Editor: John Matthews  
Production Editor: Jeffrey Rydell  
Supplement Cover Manager: Paul Gourhan  
Supplement Cover Designer: Joanne Alexandris  
Manufacturing Buyer: Ilene Kahn



© 2004 Pearson Education, Inc.  
Pearson Prentice Hall  
Pearson Education, Inc.  
Upper Saddle River, NJ 07458

All rights reserved. No part of this book may be reproduced in any form or by any means, without permission in writing from the publisher.

Pearson Prentice Hall<sup>®</sup> is a trademark of Pearson Education, Inc.

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

**ISBN 0-13-047579-3**

Pearson Education Ltd., *London*  
Pearson Education Australia Pty. Ltd., *Sydney*  
Pearson Education Singapore, Pte. Ltd.  
Pearson Education North Asia Ltd., *Hong Kong*  
Pearson Education Canada, Inc., *Toronto*  
Pearson Educación de México, S.A. de C.V.  
Pearson Education—Japan, *Tokyo*  
Pearson Education Malaysia, Pte. Ltd.  
Pearson Education, *Upper Saddle River, New Jersey*



# CONTENTS

## 1 FIRST-ORDER DIFFERENTIAL EQUATIONS

|     |  |    |
|-----|--|----|
| 1.1 | Differential Equations and Mathematical Modeling | 1  |
| 1.2 | Integrals as General and Particular Solutions    | 5  |
| 1.3 | Slope Fields and Solution Curves                 | 9  |
| 1.4 | Separable Equations and Applications             | 16 |
| 1.5 | Linear First-Order Equations                     | 24 |
| 1.6 | Substitution Methods and Exact Equations         | 28 |
|     | Chapter 1 Review Problems                        | 35 |

## 2 MATHEMATICAL MODELS AND NUMERICAL METHODS

|     |   |    |
|-----|---|----|
| 2.1 | Population Models                       | 37 |
| 2.2 | Equilibrium Solutions and Stability     | 45 |
| 2.3 | Acceleration-Velocity Models            | 53 |
| 2.4 | Numerical Approximation: Euler's Method | 57 |
| 2.5 | A Closer Look at the Euler Method       | 62 |
| 2.6 | The Runge-Kutta Method                  | 69 |

## 3 LINEAR EQUATIONS OF HIGHER ORDER

|     |   |     |
|-----|---|-----|
| 3.1 | Introduction: Second-Order Linear Equations                             | 76  |
| 3.2 | General Solutions of Linear Equations                                   | 79  |
| 3.3 | Homogeneous Equations with Constant Coefficients                        | 83  |
| 3.4 | Mechanical Vibrations   | 87  |
| 3.5 | Nonhomogeneous Equations and the<br>Method of Undetermined Coefficients | 92  |
| 3.6 | Forced Oscillations and Resonance                                       | 98  |
| 3.7 | Electrical Circuits   | 106 |
| 3.8 | Endpoint Problems and Eigenvalues                                       | 110 |

|          |  |     |
|----------|--|-----|
| <b>4</b> | <b>INTRODUCTION TO SYSTEMS OF DIFFERENTIAL EQUATIONS</b> |     |
| 4.1      | First-Order Systems and Applications                     | 115 |
| 4.2      | The Method of Elimination                                | 120 |
| 4.3      | Numerical Methods for Systems                            | 131 |
| <b>5</b> | <b>LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS</b>          |     |
| 5.1      | Linear Systems and Matrices                              | 137 |
| 5.2      | The Eigenvalue Method for Homogeneous Linear Systems     | 142 |
| 5.3      | Second-Order Systems and Mechanical Applications         | 159 |
| 5.4      | Multiple Eigenvalue Solutions                            | 165 |
| 5.5      | Matrix Exponentials and Linear Systems                   | 173 |
| 5.6      | Nonhomogeneous Linear Systems                            | 178 |
| <b>6</b> | <b>NONLINEAR SYSTEMS AND PHENOMENA</b>                   |     |
| 6.1      | Stability and the Phase Plane                            | 184 |
| 6.2      | Linear and Almost Linear Systems                         | 188 |
| 6.3      | Ecological Applications: Predators and Competitors       | 198 |
| 6.4      | Nonlinear Mechanical Systems                             | 207 |
| 6.5      | Chaos in Dynamical Systems                               | 213 |
| <b>7</b> | <b>LAPLACE TRANSFORM METHODS</b>                         |     |
| 7.1      | Laplace Transforms and Inverse Transforms                | 219 |
| 7.2      | Transformation of Initial Value Problems                 | 222 |
| 7.3      | Translation and Partial Fractions                        | 227 |
| 7.4      | Derivatives, Integrals, and Products of Transforms       | 231 |
| 7.5      | Periodic and Piecewise Continuous Forcing Functions      | 235 |
| 7.6      | Impulses and Delta Functions                             | 242 |

**8 POWER SERIES METHODS**

|            |  |     |
|------------|--|-----|
| <b>8.1</b> | Introduction and Review of Power Series    | 247 |
| <b>8.2</b> | Series Solutions Near Ordinary Points      | 251 |
| <b>8.3</b> | Regular Singular Points                    | 257 |
| <b>8.4</b> | Method of Frobenius: The Exceptional Cases | 265 |
| <b>8.5</b> | Bessel's Equation                          | 270 |
| <b>8.6</b> | Applications of Bessel Functions           | 273 |

**9 FOURIER SERIES METHODS**

|            |   |     |
|------------|---|-----|
| <b>9.1</b> | Periodic Functions and Trigonometric Series             | 277 |
| <b>9.2</b> | General Fourier Series and Convergence                  | 283 |
| <b>9.3</b> | Fourier Sine and Cosine Series                          | 290 |
| <b>9.4</b> | Applications of Fourier Series                          | 297 |
| <b>9.5</b> | Heat Conduction and Separation of Variables             | 300 |
| <b>9.6</b> | Vibrating Strings and the One-Dimensional Wave Equation | 303 |
| <b>9.7</b> | Steady-State Temperature and Laplace's Equation         | 306 |

**10 EIGENVALUES AND BOUNDARY VALUE PROBLEMS**

|             |   |     |
|-------------|---|-----|
| <b>10.1</b> | Sturm-Liouville Problems and Eigenfunction Expansions | 312 |
| <b>10.2</b> | Applications of Eigenfunction Series                  | 319 |
| <b>10.3</b> | Steady Periodic Solutions and Natural Frequencies     | 324 |
| <b>10.4</b> | Cylindrical Coordinate Problems                       | 332 |
| <b>10.5</b> | Higher-Dimensional Phenomena                          | 339 |

**APPENDIX**

|                                       |     |
|---------------------------------------|-----|
| Existence and Uniqueness of Solutions | 340 |
|---------------------------------------|-----|



## PREFACE

This is a solutions manual to accompany the textbook **DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS: Computing and Modeling** (3rd edition, 2004) by C. Henry Edwards and David E. Penney. We include solutions to most of the odd-numbered problems in the text.

Our goal is to support learning of the subject of elementary differential equations in every way that we can. We therefore invite comments and suggested improvements for future printings of this manual, as well as advice regarding features that might be added to increase its usefulness in subsequent editions. Additional supplementary material can be found at our textbook Web site listed below.

Henry Edwards & David Penney

`hedwards@math.uga.edu`

`dpenney@math.uga.edu`

`www.prenhall.com/edwards`



# CHAPTER 1

## FIRST-ORDER DIFFERENTIAL EQUATIONS

### SECTION 1.1

#### DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING

The main purpose of Section 1.1 is simply to introduce the basic notation and terminology of differential equations, and to show the student what is meant by a solution of a differential equation. Also, the use of differential equations in the mathematical modeling of real-world phenomena is outlined.

Problems 1–12 are routine verifications by direct substitution of the suggested solutions into the given differential equations. We include here just some typical examples of such verifications.

3. If  $y_1 = \cos 2x$  and  $y_2 = \sin 2x$ , then  $y_1' = -2 \sin 2x$  and  $y_2' = 2 \cos 2x$  so

$$y_1'' = -4 \cos 2x = -4 y_1 \quad \text{and} \quad y_2'' = -4 \sin 2x = -4 y_2.$$

$$\text{Thus } y_1'' + 4 y_1 = 0 \text{ and } y_2'' + 4 y_2 = 0.$$

5. If  $y = e^x - e^{-x}$ , then  $y' = e^x + e^{-x}$  so  $y' - y = (e^x + e^{-x}) - (e^x - e^{-x}) = 2e^{-x}$ . Thus  $y' = y + 2e^{-x}$ .

11. If  $y = y_1 = x^{-2}$  then  $y' = -2x^{-3}$  and  $y'' = 6x^{-4}$ , so

$$x^2 y'' + 5x y' + 4y = x^2 (6x^{-4}) + 5x (-2x^{-3}) + 4(x^{-2}) = 0.$$

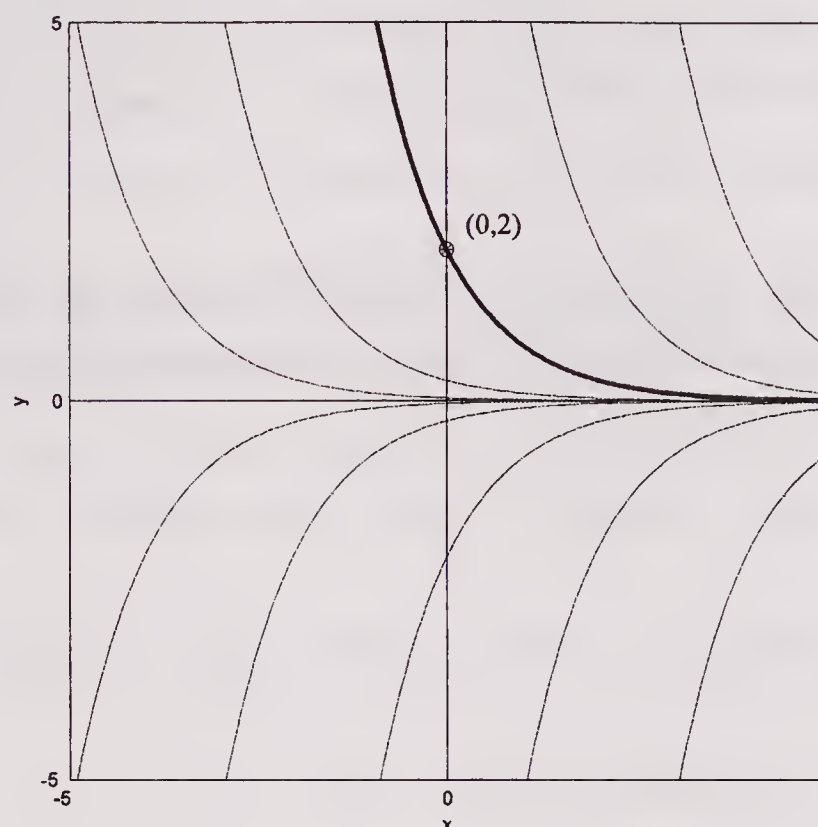
$$\text{If } y = y_2 = x^{-2} \ln x \text{ then } y' = x^{-3} - 2x^{-3} \ln x \text{ and } y'' = -5x^{-4} + 6x^{-4} \ln x, \text{ so}$$

$$\begin{aligned} x^2 y'' + 5x y' + 4y &= x^2 (-5x^{-4} + 6x^{-4} \ln x) + 5x (x^{-3} - 2x^{-3} \ln x) + 4(x^{-2} \ln x) \\ &= (-5x^{-2} + 5x^{-2}) + (6x^{-2} - 10x^{-2} + 4x^{-2}) \ln x = 0. \end{aligned}$$

13. Substitution of  $y = e^{rx}$  into  $3y' = 2y$  gives the equation  $3r e^{rx} = 2e^{rx}$  that simplifies to  $3r = 2$ . Thus  $r = 2/3$ .
15. Substitution of  $y = e^{rx}$  into  $y'' + y' - 2y = 0$  gives the equation  $r^2 e^{rx} + r e^{rx} - 2e^{rx} = 0$  that simplifies to  $r^2 + r - 2 = (r+2)(r-1) = 0$ . Thus  $r = -2$  or  $r = 1$ .

The verifications of the suggested solutions in Problems 17–26 are similar to those in Problems 1–12. We illustrate the determination of the value of  $C$  only in some typical cases. However, we illustrate typical solution curves for each of these problems.

17.  $C = 2$



19. If  $y(x) = C e^x - 1$  then  $y(0) = 5$  gives  $C - 1 = 5$ , so  $C = 6$ .

