Modern Particle Physics
Instructor's Manual
Version 1.03

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Preface to the instructor's manual

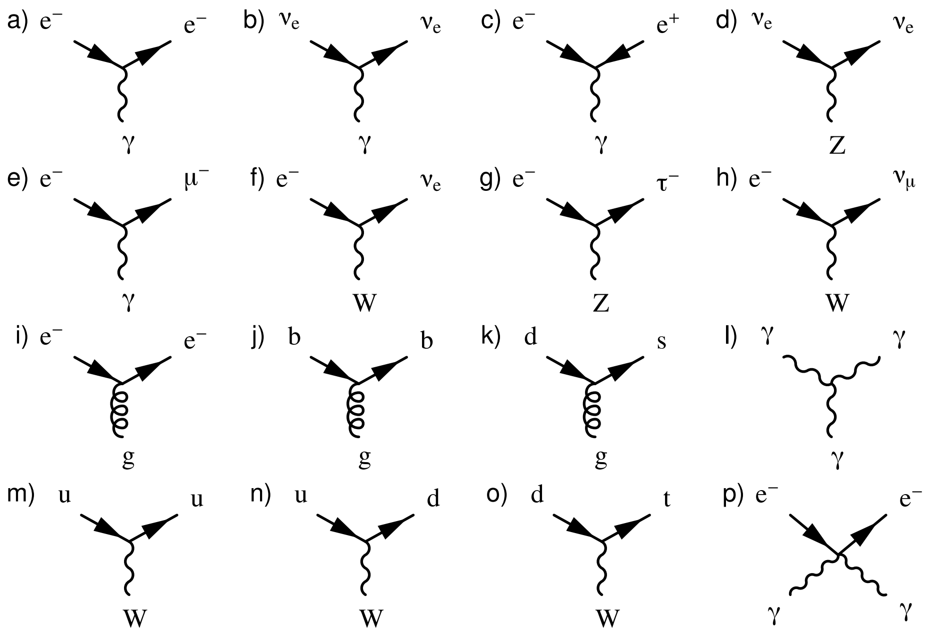
The first version of the Instructor's manual to Modern Particle Physics contains fully-worked solutions to all the problems in Chapters 1–18 of the main text. This document has not been proof-read to the extent of the main text, so I apologies in advance for any errors. Many of the problems have been used in the course that taught for a number of years, so these are battle-hardened. For new questions, introduced to address specific points in the text (particularly in the later chapters), there are a couple issues which have been noted in the solutions.

In some cases there may be more elegant approaches to the problems, the intention was to keep the solutions as straightforward as possible. Comments and suggestions are always welcome.

Mark Thomson, Cambridge, December 14th 2013

1 Introduction

1.1 Feynman diagrams are constructed out of the Standard Model vertices shown in Figure 1.4. Only the weak charged-current ($W^{\pm}$) interaction can change the flavour of the particle at the interaction vertex. Explaining your reasoning, state whether each of the sixteen diagrams below represents a valid Standard Model vertex.



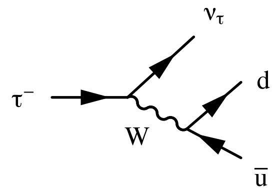
The purpose of this question is to get students to understand the that (with the exception of gauge boson triple and quartic coupling) all Feynman diagrams are built out of the Standard Model three-point vertices of Figure 1.4. Of the sixteen vertices in this question, the only valid Standard Model vertices are: a), d), f), j), n) and o). The other diagrams are forbidden for the following reasons:

- b) The electron neutrino is neutral and therefore does not couple to the gauge boson of the electromagnetic interaction;
- c) This diagram violates both charge conservation and has the effect of turning a particle into an antiparticle (the arrows on the electron lines both point towards the vertex);

e) The electron magnetic interaction does not change flavour, and hence a diagram coupling an electron to a muon is not allowed;
g) The weak neutral current also does not change flavour and hence this diagram is forbidden;
h) The weak charged current does change flavour, but *by definition* only couple together leptons with the corresponding neutrino, hence this diagram which couples together an electron and a muon neutrino is not allowed.
i) The electron does not carry the colour charge of the strong interaction - it is colour neutral - and hence the electron does not participate in the strong interaction;
k) The strong interaction does not change flavour and hence a coupling between a down-quark and a strange-quark is forbidden;
l) In the Standard Model there is no three-photon vertex;
m) Since W bosons are charged, the weak charged current must change flavour;
p) There is no Standard Model vertex couping two fermion lines to two boson lines - all fermion vertices involve a coupling to a single gauge boson.

1.2 Draw the Feynman diagram for $\tau^- \to \pi^- \nu_\tau$ (the π^- is the lightest $d\bar{u}$ meson).

Since the decay involves a change of flavour it can only be a weak charged-current interaction ($W^\pm$):



1.3 Explain why it is *not* possible to construct a valid Feynman diagram using the Standard Model vertices for the following processes:
a) $\mu^- \to e^+e^-e^+$,
b) $\nu_\tau + p \to \mu^- + n$,
c) $\nu_\tau + \bar{p} \to \tau^+ + n$,
d) $\pi^+(u\bar{d}) + \pi^-(d\bar{u}) \to n(udd) + \pi^0(u\bar{u})$.

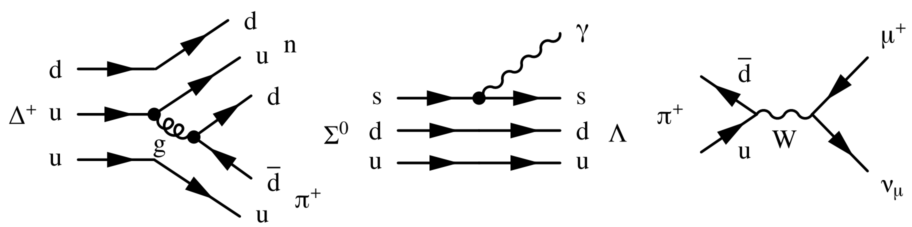
a) The process $\mu^- \to e^+e^-e^+$ would require a change of flavour. There is no problem with producing a particle and its antiparticle (here the e^+e^-) and therefore the required flavour change is $\mu^- \to e^-$ and there is no corresponding Standard Model vertex and the process cannot occur. This used to be referred to as conservation of muon and electron numbers. However, with the discovery of neutrino oscillations this concept is obsolete.

b) Two changes of flavour are required $u \to d$ and $\nu_\tau \to \mu^-$. Whilst the first of these flavour changes can be achieved by a charged-interaction vertex, the second cannot. Leptons only couple to the corresponding weak eigenstate neutrino flavour. In addition, electric charge is not conserved.
c) This process requires a vertex that has the effect $\nu_\tau \to \tau^+$, i.e. turning a particle into an antiparticle. No such vertices exist.
d) Here the net number of particles − antiparticles changes. This can not happen because all Standard Model vertices involving fermions are three point interactions with a single boson, as a consequence the net number of particles − antiparticles in the Universe is constant.

1.4 Draw the Feynman diagrams for the decays:
a) $\Delta^+(uud) \to n(udd)\,\pi^+(u\bar{d})$,
b) $\Sigma^0(uds) \to \Lambda(uds)\,\gamma$,
c) $\pi^+(u\bar{d}) \to \mu^+\nu_\mu$,

and place them in order of increasing lifetime.

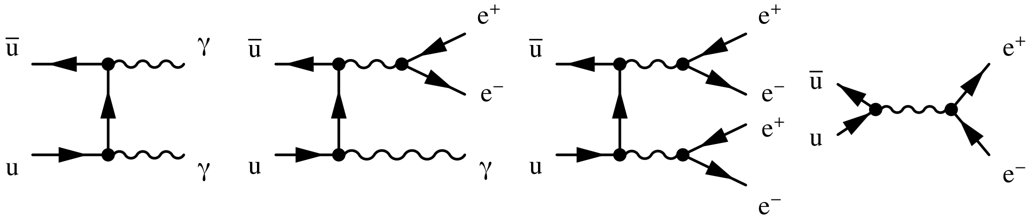
All other things being equal, strong decays will dominate over EM decays, and EM decays will dominate over weak decays. So here the order is a), b), c) with the Feynman diagrams below.



1.5 Treating the π^0 as a $u\bar{u}$ bound state, draw the Feynman diagrams for:
a) $\pi^0 \to \gamma\gamma$,
b) $\pi^0 \to \gamma e^+e^-$,
c) $\pi^0 \to e^+e^-e^+e^-$,
d) $\pi^0 \to e^+e^-$.

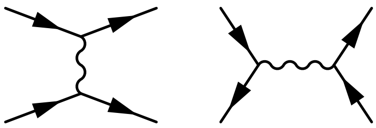
By considering the number of QED vertices present in each decay, *estimate* the relative decay rates taking $\alpha = 1/137$.

The observed branching ratios are $BR(\pi^0 \to \gamma\gamma) = 98.8\%$, $BR(\pi^0 \to \gamma e^+e^-) = 1.2\%$, $BR(\pi^0 \to \gamma e^+e^-e^+e^-) \sim 3\times10^{-5}$ and $BR(\pi^0 \to e^+e^-) = 6\times10^{-8}$. By counting the number of QED vertices it might be expected that the matrix elements for the processes are: a) $\mathcal{M} \propto e^2$; b) $\mathcal{M} \propto e^3$; c) $\mathcal{M} \propto e^4$; and d) $\mathcal{M} \propto e^2$. Consequently, one would expect the branching ratios to be proportional to a) $|\mathcal{M}|^2 \propto \alpha^2$; b) $|\mathcal{M}|^2 \propto \alpha^3$; c) $|\mathcal{M}|^2 \propto \alpha^4$; and d) $|\mathcal{M}|^2 \propto \alpha^2$. Relative to the dominant $\pi^0 \to \gamma\gamma$ decay modes it might be expected that $BR(\pi^0 \to \gamma e^+e^-) \sim O(10^{-2})$, and $BR(\pi^0 \to e^+e^-e^+e^-) \sim \mathcal{O}(10^{-4})$, in reasonable agreement with the observed values.



The observed branching ratio to e^+e^- is much smaller than that predicted from simple vertex counting (the contribution from this Feynman diagram is heavily helicity suppressed, see for example 11). This is an important point, simple vertex factor counting only addresses one of the contributions to the matrix element squared, other factors may be just as important.

1.6 Particle interactions fall into two main categories, scattering processes and annihilation processes. as indicated by the Feynman diagrams below.

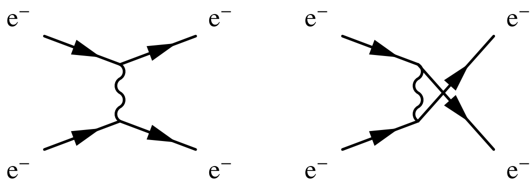


Draw the lowest-order Feynman diagrams for the scattering and/or annihilation processes:

a) $e^-e^- \to e^-e^-$,
b) $e^+e^- \to \mu^+\mu^-$,
c) $e^+e^- \to e^+e^-$,
d) $e^-\nu_e \to e^-\nu_e$,
e) $e^-\overline{\nu}_e \to e^-\overline{\nu}_e$.

In some cases there may be more than one lowest-order diagram.

a) For this scattering process there are two diagrams, the *u*-channel diagram has to be included since there are identical particles in the final state (the exchanged gauge boson could also be a Z or even the H):



b) Here there is just the *s*-channel annihilation diagram.