Solutions Manual to accompany

AN INTRODUCTION TO MECHANICS

2nd edition

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CONTENTS

case 1:
horizontal equation of motion:

$$\frac{Mv^2}{R} = N\sin\theta - f\cos\theta$$

The maximum friction force is μN.

$$\frac{Mv^2}{R} \geq N(\sin\theta - \mu\,\cos\theta)$$
$$\frac{Mv2_{min}}{R} = N(\sin\theta - \mu\,\cos\theta) \quad (1)$$

There is no vertical acceleration if the car is not sliding, so the vertical equation of motion is $N\cos\theta + f\sin\theta - Mg = 0$. In the limit where $f = \mu N$

$$Mg = N(\cos\theta + \mu\,\sin\theta) \quad (2)$$

Dividing Eq. (1) by Eq. (2),

$$\frac{v_{min}^2}{Rg} = \frac{\sin\theta - \mu\,\cos\theta}{\cos\theta + \mu\,\sin\theta} \implies v_{min} = \sqrt{Rg\left(\frac{\sin\theta - \mu\,\cos\theta}{\cos\theta + \mu\,\sin\theta}\right)}$$

case 2:
Proceeding as before,

$$M\frac{v^2}{R} \leq N\sin\theta + f\cos\theta$$
$$M\frac{v_{max}^2}{R} = N(\sin\theta + \mu\,\cos\theta) \quad (3)$$

vertical equation of motion:

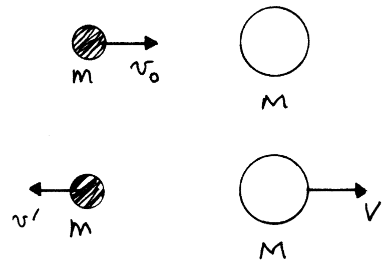
$$0 = N\cos\theta - f\sin\theta - Mg = N(\cos\theta - \mu\,\sin\theta) \quad (4)$$

Dividing Eq. (3) by Eq. (4) leads to

$$v_{max} = \sqrt{Rg\left(\frac{\sin\theta + \mu\,\cos\theta}{\cos\theta - \mu\,\sin\theta}\right)}$$

6.7 Proton collision

The proton has mass m, and the unknown particle has mass M. The upper sketch is before the collision, and the lower sketch is after the collision. Both momentum P and mechanical energy (kinetic energy K) are conserved in the elastic collision.



$$P_f = MV - mv' = P_i = mv_0 \implies v_0 = \frac{M}{m}V - v' \quad (1)$$
$$K_f = \frac{1}{2}MV^2 + \frac{1}{2}mv'^2 = K_i = \frac{1}{2}mv_0^2 \implies v_0^2 = \frac{M}{m}V^2 + v'^2 \quad (2)$$
$$E_f = \frac{1}{2}mv'^2 = \frac{4}{9}\left(\frac{1}{2}mv_0^2\right) \implies v' = \frac{2}{3}v_0 \quad (3)$$

Using Eqs. (1) and (3),

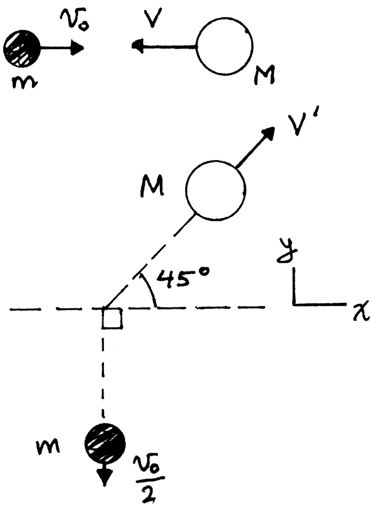
$$V = \frac{5}{3}\frac{m}{M}v_0 \quad (4)$$

Using Eqs. (3) and (4) in Eq. (2),

$$v_0^2 = \frac{M}{m}\frac{25}{9}\left(\frac{m}{M}\right)^2 v_0^2 + \frac{4}{9}v_0^2 \implies \frac{5}{9} = \frac{25}{9}\frac{m}{M} \implies M = 5m$$

6.8 Collision of m and M

The upper sketch shows the system before the collision, and the lower sketch after the collision. Both momentum **P** and mechanical energy (kinetic energy K) are conserved in the elastic collision. **P** has both x and y components.



$$P_{fx} = \frac{MV'}{\sqrt{2}} = P_{ix} = mv_0 - MV$$
$$P_{fy} = \frac{MV'}{\sqrt{2}} - \frac{mv_0}{2} = P_{iy} = 0$$
$$mv_0 - MV = \frac{MV'}{\sqrt{2}} \quad (1)$$
$$0 = \frac{MV'}{\sqrt{2}} - \frac{mv_0}{2} \implies V' = \frac{1}{\sqrt{2}}\frac{m}{M}v_0 \quad (2)$$

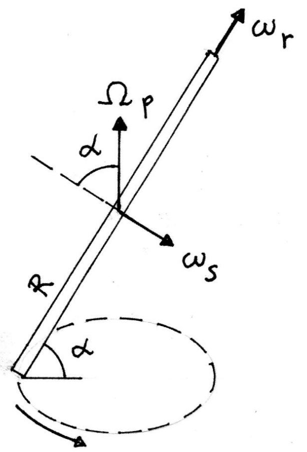
From Eqs. (1) and (2)

$$V = \frac{1}{2}\frac{m}{M}v_0 \quad (3)$$

continued next page $\implies$

8.12 *Euler's disk*

The contact point moves on the surface in a circle of radius $R\cos\alpha$, with speed $V = (R\cos\alpha)(\Omega_p$. The disk is assumed to roll without slipping, so $R\omega_s = V = (R\cos\alpha)\Omega_p$.

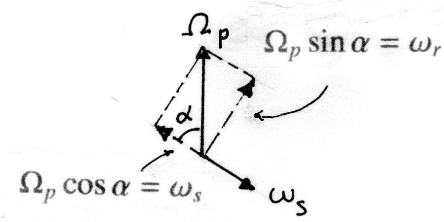


equations of motion:

$$0 = N - Mg \implies N = Mg$$

$$f = \frac{MV^2}{R\cos\alpha} = \frac{M(R\cos\alpha)^2\,\Omega_p^2}{R\cos\alpha} = MR\cos\alpha\,\Omega_p^2$$

The total angular velocity is $\mathbf{\Omega}_p + \boldsymbol{\omega}_s = \boldsymbol{\omega}_r$. As shown in the sketches, $\boldsymbol{\omega}_r$ lies along the axis from the contact point to the center of mass. The moment of inertia along this axis is



$$I_\perp = \frac{1}{2}I_0 = \frac{1}{4}MR^2$$

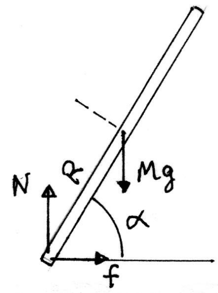
The spin angular momentum is

$$L_s = I_\perp\omega_r = \frac{1}{4}MR^2\Omega_p\sin\alpha$$

The horizontal component of the spin angular momentum is

$$L_h = L_s\cos\alpha = \frac{1}{4}MR^2\cos\alpha\sin\alpha\,\Omega_p$$

torque about the cm (positive is into the paper):

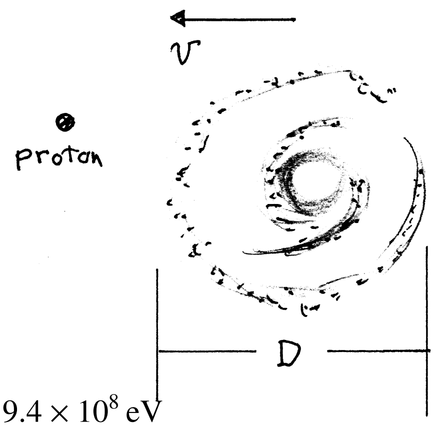


force diagram

$$\begin{aligned}\tau_{cm} &= NR\cos\alpha - fR\sin\alpha = MgR\cos\alpha - MR^2\cos\alpha\sin\alpha\,\Omega_p^2\\ &= MR\cos\alpha\left(g - R\sin\alpha\,\Omega_p^2\right)\end{aligned}$$

13 RELATIVISTIC DYNAMICS

13.1 *Energetic proton*



(a) In a frame moving with the proton, the galaxy is approaching at speed v and has thickness $D = D_0/\gamma$. The proton has such high energy that v is very nearly c, to the accuracy of this solution. The time T to traverse the galaxy is

$$T = \frac{D}{v} = \frac{D_0}{\gamma v} \simeq \frac{D_0}{\gamma c}$$

$$E = \gamma m_0 c^2 \implies \gamma = \frac{E}{m_0 c^2}$$

$$m_0 c^2 = \left(1.67 \times 10^{-27}\,\text{kg}\right)\left(3 \times 10^8\,\text{m/s}\right)^2 \left(\frac{1\,\text{eV}}{1.6 \times 10^{-19}\,\text{J}}\right) = 9.4 \times 10^8\,\text{eV}$$

$$\gamma = \frac{3 \times 10^{20}\,\text{eV}}{9.4 \times 10^8\,\text{eV}} = 3.2 \times 10^{11}$$

$$D_0 = (10^5\,\text{light years})(3 \times 10^8\,\text{m/s})\left(\frac{3 \times 10^7\,\text{s}}{1\,\text{year}}\right) = 9 \times 10^{20}\,\text{m}$$

$$T = \frac{9 \times 10^{20}\,\text{m}}{(3 \times 10^{11})(3 \times 10^8\,\text{m/s})} = 10\,\text{s}$$

The photon is traveling at the speed of light, so $\gamma \to \infty$, and $T_{photon} = 0$.

(b)

$$E_{baseball} = \frac{1}{2}Mv^2 = \frac{1}{2}(0.145\,\text{kg})\left[\left(\frac{100\,\text{miles}}{1\,\text{hour}}\right)\left(\frac{1610\,\text{m}}{1\,\text{mile}}\right)\left(\frac{1\,\text{hour}}{3600\,\text{s}}\right)\right]^2 = 145\,\text{J}$$

$$E_{proton} = (3 \times 10^{20}\,\text{eV})\left(\frac{1.6 \times 10^{-19}\,\text{J}}{1\,\text{eV}}\right) = 48\,\text{J}$$