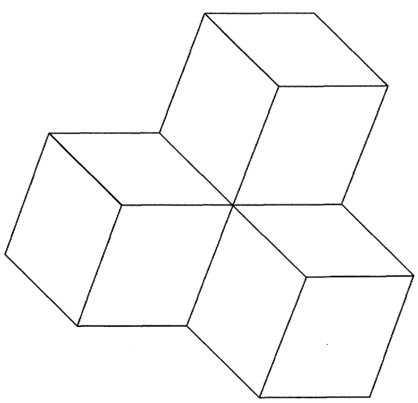
Instructor's Solutions Manual for

ELEMENTARY Linear Algebra

Stanley I. Grossman

Fifth Edition



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Preface

This Instructor's Solutions Manual is an ancillary for the fifth edition of Grossman's *Elementary Linear Algebra*. It contains detailed solutions to all problems in the text—including the MATLAB and graphing calculator problems—and in the Applications Supplement. Below is an overview of all the ancillaries to accompany the main text.

Applications Supplement	• one chapter each on linear programming and on Markov chains and game theory • available packaged with the text or for separate purchase • numerous examples and problems • answers to odd-numbered problems are at the back of the Applications Supplement
Student Solutions Manual	• complete solutions to all the odd-numbered problems in the text and the Applications Supplement
MATLAB Manual: Computer Laboratory Exercises and M-file disk	• computer laboratory exercises and applications using MATLAB. Each section lists objectives, prerequisites, and MATLAB features before the lab exercise is presented. The student is then encouraged to apply concepts interactively and create an edited diary session. An M-file disk containing programs of selected applications in the manual is available free upon request from The MathWorks, Inc., in either Mac or PC versions.
Elementary Linear Algebra Toolbox (M-file disk)	• MATLAB programs that accompany the main text in either PC or Mac version are available free upon request from The MathWorks, Inc.
HP-48G/GX Calculator Manual	• calculator enhancement for science and engineering mathematics using the high-level Hewlett-Packard calculator

Acknowledgments

This Instructor's Solutions Manual has been prepared with the help of many people. The solutions to the fourth edition problems were prepared by Rick Miranda of Colorado State University, with the assistance of Howard Thompson and John Symms. These provide the basis for much of the present work. Andy Demetre prepared the solutions for the new problems in the main body of the text. Fred Gylys-Colwell developed the solutions for the MATLAB problems in Chapters 1 and 4. David Ragozin provided the solutions for the MATLAB problems in Chapters 5 and 6, the CALCULATOR box solutions for the TI-85, and the editorial changes and updated solutions found throughout the rest of the manual. Michael Ragozin assisted with the TI-85 solutions.

Mary Sheets produced the TEX files for the manual. The new figures were produced by Fred Gylys-Colwell and David Ragozin using MATLAB, S-Plus, and TI-85 Graph Link.

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March 1994

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33. Let c be the number of chairs and t the number of tables produced each day. We have $8 \cdot 12 = 96$ labor hours per day in the machine shop. Hence, for the machine shop we must have $\frac{384}{17} \cdot c + \frac{240}{17} \cdot t = 96$. Similarly, $\frac{480}{17} \cdot c + \frac{640}{17} \cdot t = 8 \cdot 20 = 160$ for the assembly and finishing division. Write this system as $A\mathbf{x} = b$ where $A = \begin{pmatrix} \frac{384}{17} & \frac{240}{17} \\ \frac{480}{17} & \frac{640}{17} \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} c \\ t \end{pmatrix}$, and $b = \begin{pmatrix} 96 \\ 160 \end{pmatrix}$. Since $\det A = \frac{130,560}{289} \neq 0$, then $\mathbf{x} = A^{-1}b = \frac{289}{130,560} \begin{pmatrix} \frac{640}{17} & -\frac{240}{17} \\ -\frac{480}{17} & \frac{384}{17} \end{pmatrix} \begin{pmatrix} 96 \\ 160 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Hence, 3 chairs and 2 tables can be produced each day.

34. Let l be the amount of love potion and c be the amount of cold remedy needed. The witch wants to find $\mathbf{x} = \begin{pmatrix} l \\ c \end{pmatrix}$ such that $A\mathbf{x} = \mathbf{b}$ where $A = \begin{pmatrix} 3\frac{1}{13} & 5\frac{5}{13} \\ 2\frac{2}{13} & 10\frac{10}{13} \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$. Since $\det A = \frac{1}{169}(40 \cdot 140 - 28 \cdot 70) = \frac{280}{13} \neq 0$, then $\mathbf{x} = A^{-1}\mathbf{b} = \frac{13}{280} \begin{pmatrix} \frac{140}{13} & -\frac{70}{13} \\ -\frac{28}{13} & \frac{40}{13} \end{pmatrix} \begin{pmatrix} 10 \\ 14 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 1 \end{pmatrix}$. Hence, $1\frac{1}{2}$ batches of love potion and 1 batch of cold remedy are needed.

35. The farmer needs $A\mathbf{x} = \mathbf{b}$ where $A = \begin{pmatrix} 0.10 & 0.12 \\ 0.15 & 0.08 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Since $\det A = -0.01 \neq 0$ then $\mathbf{x} = A^{-1}\mathbf{b} = -100 \begin{pmatrix} 0.08 & -0.12 \\ -0.15 & 0.10 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. Thus, 4 units of type A and 5 units of type B are needed.

36. (a) 0.293 (b) $200,000 \cdot 0.293 = 58,600$ (c) 0
(d) $50,000 \cdot 0.044 = 2,200$

37. (a) technology matrix $A = \begin{pmatrix} 0.293 & 0 & 0 \\ 0.014 & 0.207 & 0.017 \\ 0.044 & 0.010 & 0.216 \end{pmatrix}$

Leontief matrix $= I - A = \begin{pmatrix} 0.707 & 0 & 0 \\ -0.014 & 0.793 & -0.017 \\ -0.044 & -0.010 & 0.784 \end{pmatrix}$

(b) $(I - A)^{-1} = \begin{pmatrix} 1.414 & 0 & 0 \\ 0.027 & 1.261 & 0.027 \\ 0.080 & 0.016 & 1.276 \end{pmatrix}$ $\quad \mathbf{x} = (I - A)^{-1} \begin{pmatrix} 13,213 \\ 17,597 \\ 1,786 \end{pmatrix} = \begin{pmatrix} 18,689 \\ 22,598 \\ 3,615 \end{pmatrix}$

Section 2.4

1. $\det A = 4 \qquad \operatorname{adj} A = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 1/2 & -1/2 \\ -1/4 & 3/4 \end{pmatrix}$

2. $\det A = 0$; A is not invertible

3. $\det A = -1 \qquad \operatorname{adj} A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

4. $\det A = -8 \qquad \operatorname{adj} A = \begin{pmatrix} -13 & 4 & 1 \\ 15 & 4 & -3 \\ -10 & 0 & 2 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 13/8 & -1/2 & -1/8 \\ -15/8 & 1/2 & 3/8 \\ 5/4 & 0 & -1/4 \end{pmatrix}$

5. $\det A = -12 \qquad \operatorname{adj} A = \begin{pmatrix} -4 & 3 & 2 \\ 0 & -3 & -6 \\ 0 & -3 & 6 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 1/3 & -1/4 & -1/6 \\ 0 & 1/4 & 1/2 \\ 0 & 1/4 & -1/2 \end{pmatrix}$

6. $\det A = 1 \qquad \operatorname{adj} A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = A^{-1}$

7. $\det A = -1 \qquad \operatorname{adj} A = \begin{pmatrix} 0 & -1 & 1 \\ -2 & 2 & 1 \\ 1 & -1 & -1 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

8. $\det A = -8 \qquad \operatorname{adj} A = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 3 & -6 \\ 2 & -2 & -4 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 3/8 & 1/8 & -1/4 \\ -1/8 & -3/8 & 3/4 \\ -1/4 & 1/4 & 1/2 \end{pmatrix}$

9. $\det A = 0$; A is not invertible

10. $\det A = 0$; A is not invertible

11. $\det A = -9 \qquad \operatorname{adj} A = \begin{pmatrix} -21 & 3 & 3 & 6 \\ -4 & 1 & 4 & -1 \\ 1 & 2 & -1 & -2 \\ 15 & -6 & -6 & -3 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 7/3 & -1/3 & -1/3 & -2/3 \\ 4/9 & -1/9 & -4/9 & 1/9 \\ -1/9 & -2/9 & 1/9 & 2/9 \\ -5/3 & 2/3 & 2/3 & 1/3 \end{pmatrix}$

12. $\det A = -1 \qquad \operatorname{adj} A = \begin{pmatrix} 0 & -1 & 0 & -2 \\ -1 & 1 & 2 & -2 \\ 0 & -1 & -3 & 3 \\ 2 & -2 & -3 & 2 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 0 & 1 & 0 & 2 \\ 1 & -1 & -2 & 2 \\ 0 & 1 & 3 & -3 \\ -2 & 2 & 3 & -2 \end{pmatrix}$

13. By theorem 2.2.3, $\det A = \det A^t$. Hence, $\det A$ is nonzero if and only if $\det A^t$ is nonzero. By theorem 4, A is invertible if and only if A^t is invertible.

14. $\det A = 3 \qquad A^{-1} = \frac{1}{3}\begin{pmatrix} 5 & -1 \\ -2 & 1 \end{pmatrix} \qquad \det A^{-1} = 1/3$

15. $\det A = -28 \qquad A^{-1} = -\frac{1}{28}\begin{pmatrix} -2 & -2 & -9 \\ 20 & -8 & 6 \\ -2 & -2 & 5 \end{pmatrix} \qquad \det A^{-1} = -\frac{1}{28}$

16. $\begin{vmatrix} \alpha & -3 \\ 4 & 1-\alpha \end{vmatrix} = \alpha - \alpha^2 + 12 = 0$. If $\alpha = 4$ or -3 then the matrix is not invertible.

17. $\begin{vmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{vmatrix} = \begin{vmatrix} 0 & 3\alpha-1 & 4\alpha+1 \\ 1 & 2 & 3 \\ 0 & 3\alpha-1 & 4\alpha+1 \end{vmatrix} = 0$ $(R_1 = R_3)$. Hence, for all values of α, the matrix is not invertible.

CALCULATOR SOLUTIONS 4.10

Problems 13-16 ask you to find the linear regression line for some x-y data pairs and then problems 17-20 request the quadratic regression curve for the same data; each type is to be calculated to eight significant digits. To have the results listed to the requested accuracy we place the calculator in scientific mode with 8 digits to the right of the decimal place by using the **MODE** menu or by keying in `SCI:FIX 8` [ENTER] . Next we carryout the desired regression calculation using the functions **LINR** and **P2REG** from the **STAT** menu, both of which have x-list and y-list arguments.

Two different approaches to entering the data and carrying out the calculations can be followed.

Either we can do the (x,y) pair data entry together from the **STAT** menu and then perform the regression calculations as suggested in the text:

1. We enter the **STAT** menu and prepare to enter the lists via:

   [STAT] [F2] **<EDIT>**

2. We name the lists (with the problem number since we'll have to reuse each one) via:

   `X41013` [ENTER] `Y41013` [ENTER]

3. We then enter the (x,y) points in order by

   `57` [ENTER] `84` [ENTER] `43` [ENTER] `91` [ENTER] `71` [ENTER] `36` [ENTER] `83` [ENTER] `24` [ENTER] `108` [ENTER] `15` [ENTER] `141` [ENTER] `8` [ENTER] . (We can type [EXIT] at this point to indicate we are done entering data, althogh this is not necessary.)

4. Then we calculate the linear regression equation:

   (a) We go to the **STAT CALC** menu by [2nd] [CALC] (if we stopped the previous step without an [EXIT]) or by [STAT] [F1] **<CALC>** (if we used [EXIT]).

   (b) [ENTER] [ENTER] (to accept the x-list and y-list entered previously; or insert a different x-list and/or y-list name before each [ENTER] .

   (c) [F2] **<LINR>** to calculate and display the regression coefficents for the line $y = a + bx$, or [MORE] [F1] **<P2REG>** to calculate and display the regression coefficients $\{c_2, c_1, c_0\}$ for the 2nd degree polynomial $y = c_2x^2 + c_1x + c_0$ (yes the cofficients for the polynomial come out in this (reversed) order).

Or we can enter the x-data into a list and the y-data (say for Problem 13) into a list from the [2nd] [LIST] menu by `{57,43,71,83,108,141}` [STO▷] `X41013` [ENTER] and `{84,91,36,24,15 8}` [STO▷] `Y41013` [ENTER] . Then compute the linear regression equation by literally keying in `LINR(X41013,Y41013)` [ENTER] . To see the coefficients of the linear regression line $y = a + bx$ we must enter `SHWST` [ENTER] , which displays the last computed **STAT CALC** result. For the 2'nd degree polynomial regression equation from this data (i.e. for Problem 17) instead of `LINR` we key in `P2REG(X41013,Y41013):PRegC` [ENTER] which calculates and displays the coefficients for the quadratic regression curve.

Although you are not asked to get the graphs of the regression curves, it is very easy to do and very helpful in assesing the reasonableness of the least squares fitting that has been done. After you perform the regression step above, you can look at the graphs by doing the following steps:

G1. Go to the **GRAPH RANGE** menu by entering [GRAPH] [F2] **<RANGE>** and establish a reasonable graphing range which encloses the min and max of the x-list and the y-list. For Problem 13 and 17 this might be done by entering values `xMin=0` [▾] `xMax=150` [▾] `yMin=0` [▾] `yMax=100`. The values you enter here should be nice values slightly outside the x-list and y-list limits. (If you want to have axes with tick marks you will have to set `LabelOn` on the **GRAPH FORMT** menu, and you should set `xScl=10` (or 25) and `yScl=10`(or25) during the **RANGE** setting operation. The choices for scales should be chosen to mesh

12. (a)

```
>>  A = [-.01969633   .01057339  -.005030409;...
>>        .01057339   .008020058 -.006818069;...
         -.005030409 -.006818069  .01158627 ];
>>  [V,D]=eig(A); maxext=max(diag(D)) , maxcomp=min(diag(D)),
maxext =
    0.0197
maxcomp =
   -0.0235
>>  for k=1:3, if maxext==D(k,k),  MaxExt1Dir=V(:,k),end,end
MaxExt1Dir =
   -0.2655
   -0.6526
    0.7097
>>  for k=1:3, if maxcomp==D(k,k), MaxComp1Dir=V(:,k),end,end
MaxComp1Dir =
    0.9501
   -0.3022
    0.0776

>>  A = [-.01470626   .01001909  -.004158314;...
>>        .01001901   .007722046 -.004482362;...
         -.004158314 -.004482362  .006984212];
>>  [V,D]=eig(A); maxext=max(diag(D)) , maxcomp=min(diag(D)),
maxext =
    0.0154
maxcomp =
   -0.0187
>>  for k=1:3, if maxext==D(k,k), MaxExt2Dir=V(:,k),end,end
MaxExt2Dir =
    0.3296
    0.7569
   -0.5643
>>  for k=1:3, if maxcomp==D(k,k), MaxComp2Dir=V(:,k),end,end
MaxComp2Dir =
    0.9363
   -0.3388
    0.0923
```

(b) Since [1 0 0] and any column of `V` from `eig(A)` are unit vectors as A is symmetrix, the following give the requested (bedding) angles in degrees:

```
>> acos([1 0 0]*MaxComp1Dir)*180/pi % Angle : Compression Axis - 1st A
ans =
   18.1801
>> acos([1 0 0]*MaxComp2Dir)*180/pi % Angle : Compression Axis - 2nd A
ans =
   20.5600
```

(c) The bedding angles computed in (b) were about 18° and 21°, so far from 45°.

33. Let r, e, p, and n denote the number of rings, earrings, pins, and necklaces, respectively. We want to maximize $E = 50r + 80e + 25e + 200n$ subject to the constraints: $0 \le r \le 10$, $0 \le e \le 10$, $0 \le p \le 15$, $0 \le n \le 3$, and $2r + 2e + p + 4n \le 40$. Then

r	e	p	n	s_1	s_2	s_3	s_4	s_5		
2	2	1	4	1	0	0	0	0	40	s_1
1	0	0	0	0	1	0	0	0	10	s_2
0	1	0	0	0	0	1	0	0	10	s_3
0	0	1	0	0	0	0	1	0	15	s_4
0	0	0	1	0	0	0	0	1	3	s_5
50	80	25	200	0	0	0	0	0	E	

$\rightarrow$

r	e	p	n	s_1	s_2	s_3	s_4	s_5		
0	0	0	1	0	0	0	0	1	3	n
1	0	0.5	0	0.5	0	-1	0	-2	4	r
0	1	0	0	0	0	1	0	0	10	e
0	0	1	0	0	0	0	1	0	15	s_4
0	0	-0.5	0	-0.5	1	1	0	2	6	s_5
0	0	0	0	-25	0	-30	0	-100	$E - 1,600$	

Thus with $(r, e, p, n) = (4, 10, 0, 3)$, E is maximized and $E = \$1600$. Note that the jeweler can make 2 pins instead of a ring, with the same profit, so there are more solutions.

34. Using the same notation as in #29 of Application Section 1.2, we have

x_1	x_2	x_3	s_1	s_2	s_3		
1/2	1/3	0	1	0	0	10,000	s_1
1/2	1/3	1/2	0	1	0	12,000	s_2
0	1/3	1/2	0	0	1	8,000	s_3
0.3	0.4	0.5	0	0	0	P	

$\rightarrow$

x_1	x_2	x_3	s_1	s_2	s_3		
1	0	0	0	2	-2	8,000	x_1
0	0	1	-2	2	0	4,000	x_3
0	1	0	3	-3	3	18,000	x_2
0	0	0	-0.2	-0.4	-0.6	$P - 11,600$	

As before, $(x_1, x_2, x_3) = (8000, 18000, 4000)$ and $P = \$11600$.

35. Using the same notation as in #28 of Application Section 1.2, we have

x_1	x_2	x_3	s_1		
0.5	1	2	1	80	s_1
100	150	200	0	P	

$\rightarrow$

x_1	x_2	x_3	s_1		
1	2	4	2	160	x_1
0	-50	-200	-200	$P - 16,000$	

and

x_1	x_2	x_3	s_1		
0.5	1	2	1	80	s_1
1	1	1	0	N	

$\rightarrow$

x_1	x_2	x_3	s_1		
1	2	4	2	160	x_1
0	-1	-3	-2	$N - 160$	

Hence with $(x_1, x_2, x_3) = (160, 0, 0)$, both P and N are maximized, $P = \$16000$, and $N = \$160$.

36. Note: See Application Section 1.5 for information on the method used to solve this problem. From problem 21, we want to minimize $g = 0.5y_1 + y_2$ subject to the constraints: $0.9y_1 + 0.6y_2 \ge 2$, $0.1y_1 + 0.4y_2 \ge 1$, $y_1 \ge 0$, and $y_2 \ge 0$. The dual problem is to maximize $f = 2x_1 + x_2$ subject to the constraints: $0.9x_1 + 0.1x_2 \le 0.5$, $0.6x_1 + 0.4x_2 \le 1$, $x_1 \ge 0$, and $x_2 \ge 0$. This gives

x_1	x_2	s_1	s_2		
0.9	0.1	1	0	0.5	s_1
0.6	0.4	0	1	1	s_2
2	1	0	0	f	

$\rightarrow$

x_1	x_2	s_1	s_2		
1	0	4/3	-1/3	1/3	x_1
0	1	-2	3	0	x_2
0	0	-2/3	-7/3	$f - 8/3$	