

STUDENT SOLUTIONS MANUAL TO  
ACCOMPANY JON ROGAWSKI'S

# CALCULUS

Single Variable

SECOND EDITION

Brian Bradie  
Roger Lipsett

**Student's Solutions Manual  
to accompany Jon Rogawski's**

Single Variable

# CALCULUS

SECOND EDITION

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W. H. FREEMAN AND COMPANY  
NEW YORK

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**SOLUTION** Let  $f(x) = 1/\sqrt{x}$ . Then

$$\begin{aligned}\frac{f(5+h) - f(5)}{h} &= \frac{\frac{1}{\sqrt{5+h}} - \frac{1}{\sqrt{5}}}{h} = \frac{\sqrt{5} - \sqrt{5+h}}{h\sqrt{5}\sqrt{5+h}} \\ &= \frac{\sqrt{5} - \sqrt{5+h}}{h\sqrt{5}\sqrt{5+h}} \left( \frac{\sqrt{5} + \sqrt{5+h}}{\sqrt{5} + \sqrt{5+h}} \right) \\ &= \frac{5 - (5+h)}{h\sqrt{5}\sqrt{5+h}(\sqrt{5+h} + \sqrt{5})} = -\frac{1}{\sqrt{5}\sqrt{5+h}(\sqrt{5+h} + \sqrt{5})}.\end{aligned}$$

Thus,

$$\begin{aligned}f'(5) &= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0} -\frac{1}{\sqrt{5}\sqrt{5+h}(\sqrt{5+h} + \sqrt{5})} \\ &= -\frac{1}{\sqrt{5}\sqrt{5}(\sqrt{5} + \sqrt{5})} = -\frac{1}{10\sqrt{5}}.\end{aligned}$$

In Exercises 27–44, use the limit definition to compute  $f'(a)$  and find an equation of the tangent line.

**27.**  $f(x) = 2x^2 + 10x$ ,  $a = 3$

**SOLUTION** Let  $f(x) = 2x^2 + 10x$ . Then

$$\begin{aligned}f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{2(3+h)^2 + 10(3+h) - 48}{h} \\ &= \lim_{h \rightarrow 0} \frac{18 + 12h + 2h^2 + 30 + 10h - 48}{h} = \lim_{h \rightarrow 0} (22 + 2h) = 22.\end{aligned}$$

At  $a = 3$ , the tangent line is

$$y = f'(3)(x - 3) + f(3) = 22(x - 3) + 48 = 22x - 18.$$

**29.**  $f(t) = t - 2t^2$ ,  $a = 3$

**SOLUTION** Let  $f(t) = t - 2t^2$ . Then

$$\begin{aligned}f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{(3+h) - 2(3+h)^2 - (-15)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3 + h - 18 - 12h - 2h^2 + 15}{h} \\ &= \lim_{h \rightarrow 0} (-11 - 2h) = -11.\end{aligned}$$

At  $a = 3$ , the tangent line is

$$y = f'(3)(t - 3) + f(3) = -11(t - 3) - 15 = -11t + 18.$$

**31.**  $f(x) = x^3 + x$ ,  $a = 0$

**SOLUTION** Let  $f(x) = x^3 + x$ . Then

$$\begin{aligned}f'(0) &= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^3 + h - 0}{h} \\ &= \lim_{h \rightarrow 0} (h^2 + 1) = 1.\end{aligned}$$

At  $a = 0$ , the tangent line is

$$y = f'(0)(x - 0) + f(0) = x.$$

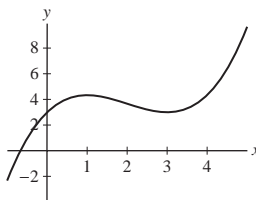
**33.**  $f(x) = x^{-1}$ ,  $a = 8$

**SOLUTION** Let  $f(x) = x^{-1}$ . Then

$$f'(8) = \lim_{h \rightarrow 0} \frac{f(8+h) - f(8)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{8+h} - \left(\frac{1}{8}\right)}{h} = \lim_{h \rightarrow 0} \frac{\frac{8-8-h}{8(8+h)}}{h} = \lim_{h \rightarrow 0} \frac{-h}{(64+8h)h} = -\frac{1}{64}$$

17.  $f'(x)$  is negative on  $(1, 3)$  and positive everywhere else.

**SOLUTION** Here is the graph of a function  $f$  for which  $f'(x)$  is negative on  $(1, 3)$  and positive elsewhere.



In Exercises 19–22, find all critical points of  $f$  and use the First Derivative Test to determine whether they are local minima or maxima.

19.  $f(x) = 4 + 6x - x^2$

**SOLUTION** Let  $f(x) = 4 + 6x - x^2$ . Then  $f'(x) = 6 - 2x = 0$  implies that  $x = 3$  is the only critical point of  $f$ . As  $x$  increases through 3,  $f'(x)$  makes the sign transition  $+$ ,  $-$ . Therefore,  $f(3) = 13$  is a local maximum.

21.  $f(x) = \frac{x^2}{x+1}$

**SOLUTION** Let  $f(x) = \frac{x^2}{x+1}$ . Then

$$f'(x) = \frac{x(x+2)}{(x+1)^2} = 0$$

implies that  $x = 0$  and  $x = -2$  are critical points. Note that  $x = -1$  is not a critical point because it is not in the domain of  $f$ . As  $x$  increases through  $-2$ ,  $f'(x)$  makes the sign transition  $+$ ,  $-$  so  $f(-2) = -4$  is a local maximum. As  $x$  increases through 0,  $f'(x)$  makes the sign transition  $-$ ,  $+$  so  $f(0) = 0$  is a local minimum.

In Exercises 23–44, find the critical points and the intervals on which the function is increasing or decreasing. Use the First Derivative Test to determine whether the critical point is a local min or max (or neither).

**SOLUTION** Here is a table legend for Exercises 23–44.

SYMBOL	MEANING
$-$	The entity is negative on the given interval.
$0$	The entity is zero at the specified point.
$+$	The entity is positive on the given interval.
$U$	The entity is undefined at the specified point.
$\nearrow$	$f$ is increasing on the given interval.
$\searrow$	$f$ is decreasing on the given interval.
$M$	$f$ has a local maximum at the specified point.
$m$	$f$ has a local minimum at the specified point.
$\neg$	There is no local extremum here.

23.  $y = -x^2 + 7x - 17$

**SOLUTION** Let  $f(x) = -x^2 + 7x - 17$ . Then  $f'(x) = 7 - 2x = 0$  yields the critical point  $c = \frac{7}{2}$ .

$x$	$(-\infty, \frac{7}{2})$	$7/2$	$(\frac{7}{2}, \infty)$
$f'$	$+$	$0$	$-$
$f$	$\nearrow$	$M$	$\searrow$

25.  $y = x^3 - 12x^2$

**SOLUTION** Let  $f(x) = x^3 - 12x^2$ . Then  $f'(x) = 3x^2 - 24x = 3x(x - 8) = 0$  yields critical points  $c = 0, 8$ .



5. A certain RNA molecule replicates every 3 minutes. Find the differential equation for the number  $N(t)$  of molecules present at time  $t$  (in minutes). How many molecules will be present after one hour if there is one molecule at  $t = 0$ ?

**SOLUTION** The doubling time is  $\frac{\ln 2}{k}$  so  $k = \frac{\ln 2}{\text{doubling time}}$ . Thus, the differential equation is  $N'(t) = kN(t) = \frac{\ln 2}{3}N(t)$ . With one molecule initially,

$$N(t) = e^{(\ln 2/3)t} = 2^{t/3}.$$

Thus, after one hour, there are

$$N(60) = 2^{60/3} = 1,048,576$$

molecules present.

7. Find all solutions to the differential equation  $y' = -5y$ . Which solution satisfies the initial condition  $y(0) = 3.4$ ?

**SOLUTION**  $y' = -5y$ , so  $y(t) = Ce^{-5t}$  for some constant  $C$ . The initial condition  $y(0) = 3.4$  determines  $C = 3.4$ . Therefore,  $y(t) = 3.4e^{-5t}$ .

9. Find the solution to  $y' = 3y$  satisfying  $y(2) = 1000$ .

**SOLUTION**  $y' = 3y$ , so  $y(t) = Ce^{3t}$  for some constant  $C$ . The initial condition  $y(2) = 1000$  determines  $C = \frac{1000}{e^6}$ . Therefore,  $y(t) = \frac{1000}{e^6}e^{3t} = 1000e^{3(t-2)}$ .

11. The decay constant of cobalt-60 is  $0.13 \text{ year}^{-1}$ . Find its half-life.

**SOLUTION** Half-life =  $\frac{\ln 2}{0.13} \approx 5.33$  years.

13. One of the world's smallest flowering plants, *Wolffia globosa* (Figure 11), has a doubling time of approximately 30 hours. Find the growth constant  $k$  and determine the initial population if the population grew to 1000 after 48 hours.

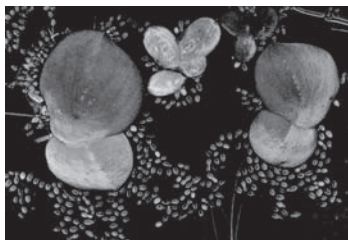


FIGURE 11 The tiny plants are *Wolffia*, with plant bodies smaller than the head of a pin.

**SOLUTION** By the formula for the doubling time,  $30 = \frac{\ln 2}{k}$ . Therefore,

$$k = \frac{\ln 2}{30} \approx 0.023 \text{ hours}^{-1}.$$

The plant population after  $t$  hours is  $P(t) = P_0e^{0.023t}$ . If  $P(48) = 1000$ , then

$$P_0e^{(0.023)48} = 1000 \Rightarrow P_0 = 1000e^{-(0.023)48} \approx 332$$

15. The population of a city is  $P(t) = 2 \cdot e^{0.06t}$  (in millions), where  $t$  is measured in years. Calculate the time it takes for the population to double, to triple, and to increase seven-fold.

**SOLUTION** Since  $k = 0.06$ , the doubling time is

$$\frac{\ln 2}{k} \approx 11.55 \text{ years}.$$

The tripling time is calculated in the same way as the doubling time. Solve for  $\Delta$  in the equation

$$\begin{aligned} P(t + \Delta) &= 3P(t) \\ 2 \cdot e^{0.06(t+\Delta)} &= 3(2e^{0.06t}) \\ 2 \cdot e^{0.06t} e^{0.06\Delta} &= 3(2e^{0.06t}) \\ e^{0.06\Delta} &= 3 \\ 0.06\Delta &= \ln 3, \end{aligned}$$

$$35. \int \frac{(6x^2 + 2) dx}{x^2 + 2x - 3}$$

**SOLUTION** Long division gives

$$\frac{6x^2 + 2}{x^2 + 2x - 3} = 6 - \frac{12x - 20}{x^2 + 2x - 3} = 6 - \frac{12x - 20}{(x + 3)(x - 1)}$$

The partial fraction decomposition of the second term is

$$\frac{12x - 20}{(x + 3)(x - 1)} = \frac{A}{x + 3} + \frac{B}{x - 1}$$

Clear fractions to get

$$12x - 20 = A(x - 1) + B(x + 3)$$

Set  $x = 1$  to get  $-8 = 4B$  so that  $B = -2$ . Set  $x = -3$  to get  $-56 = -4A$  so that  $A = 14$ , and we have

$$\begin{aligned} \int \frac{6x^2 + 2}{x^2 + 2x - 3} dx &= \int 6 - \frac{14}{x + 3} + \frac{2}{x - 1} dx = \int 6 dx - 14 \int \frac{1}{x + 3} dx + 2 \int \frac{1}{x - 1} dx \\ &= 6x - 14 \ln |x + 3| + 2 \ln |x - 1| + C \end{aligned}$$

$$37. \int \frac{10 dx}{(x - 1)^2(x^2 + 9)}$$

**SOLUTION** The partial fraction decomposition has the form:

$$\frac{10}{(x - 1)^2(x^2 + 9)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 9}.$$

Clearing denominators, we get

$$10 = A(x - 1)(x^2 + 9) + B(x^2 + 9) + (Cx + D)(x - 1)^2.$$

Setting  $x = 1$  then yields

$$10 = 0 + B(10) + 0 \quad \text{or} \quad B = 1.$$

Expanding the right-hand side, we have

$$10 = (A + C)x^3 + (1 - A - 2C + D)x^2 + (9A + C - 2D)x + (9 - 9A + D).$$

Equating coefficients of like powers of  $x$  then yields

$$A + C = 0$$

$$1 - A - 2C + D = 0$$

$$9A + C - 2D = 0$$

$$9 - 9A + D = 10$$

From the first equation, we have  $C = -A$ , and from the fourth equation we have  $D = 1 + 9A$ . Substituting these into the second equation, we get

$$1 - A - 2(-A) + (1 + 9A) = 0 \quad \text{or} \quad A = -\frac{1}{5}.$$

Finally,  $C = \frac{1}{5}$  and  $D = -\frac{4}{5}$ . The result is

$$\frac{10}{(x - 1)^2(x^2 + 9)} = \frac{-\frac{1}{5}}{x - 1} + \frac{1}{(x - 1)^2} + \frac{\frac{1}{5}x - \frac{4}{5}}{x^2 + 9}.$$

Thus,

$$\begin{aligned} \int \frac{10 dx}{(x - 1)^2(x^2 + 9)} &= -\frac{1}{5} \int \frac{dx}{x - 1} + \int \frac{dx}{(x - 1)^2} + \frac{1}{5} \int \frac{x dx}{x^2 + 9} - \frac{4}{5} \int \frac{dx}{x^2 + 9} \\ &= -\frac{1}{5} \ln |x - 1| - \frac{1}{x - 1} + \frac{1}{10} \ln |x^2 + 9| - \frac{4}{15} \tan^{-1} \left( \frac{x}{3} \right) + C. \end{aligned}$$



5. Show that the arc length of  $y = 2\sqrt{x}$  over  $[0, a]$  is equal to  $\sqrt{a(a+1)} + \ln(\sqrt{a} + \sqrt{a+1})$ . *Hint:* Apply the substitution  $x = \tan^2 \theta$  to the arc length integral.

**SOLUTION** Let  $y = 2\sqrt{x}$ . Then  $y' = \frac{1}{\sqrt{x}}$ , and

$$\sqrt{1 + (y')^2} = \sqrt{1 + \frac{1}{x}} = \sqrt{\frac{x+1}{x}} = \frac{1}{\sqrt{x}} \sqrt{x+1}.$$

Thus,

$$s = \int_0^a \frac{1}{\sqrt{x}} \sqrt{1+x} \, dx.$$

We make the substitution  $x = \tan^2 \theta$ ,  $dx = 2 \tan \theta \sec^2 \theta \, d\theta$ . Then

$$s = \int_{x=0}^{x=a} \frac{1}{\tan \theta} \sec \theta \cdot 2 \tan \theta \sec^2 \theta \, d\theta = 2 \int_{x=0}^{x=a} \sec^3 \theta \, d\theta.$$

We use a reduction formula to obtain

$$\begin{aligned} s &= 2 \left( \frac{\tan \theta \sec \theta}{2} + \frac{1}{2} \ln |\sec \theta + \tan \theta| \right) \Big|_{x=0}^{x=a} = (\sqrt{x} \sqrt{1+x} + \ln |\sqrt{1+x} + \sqrt{x}|) \Big|_0^a \\ &= \sqrt{a} \sqrt{1+a} + \ln |\sqrt{1+a} + \sqrt{a}| = \sqrt{a(a+1)} + \ln (\sqrt{a} + \sqrt{a+1}). \end{aligned}$$

In Exercises 7–10, calculate the surface area of the solid obtained by rotating the curve over the given interval about the  $x$ -axis.

7.  $y = x + 1$ ,  $[0, 4]$

**SOLUTION** Let  $y = x + 1$ . Then  $y' = 1$ , and

$$y\sqrt{1 + y'^2} = (x+1)\sqrt{1+1} = \sqrt{2}(x+1).$$

Thus,

$$SA = 2\pi \int_0^4 \sqrt{2}(x+1) \, dx = 2\sqrt{2}\pi \left( \frac{x^2}{2} + x \right) \Big|_0^4 = 24\sqrt{2}\pi.$$

9.  $y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$ ,  $[1, 2]$

**SOLUTION** Let  $y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$ . Then

$$y' = \sqrt{x} - \frac{1}{4\sqrt{x}},$$

and

$$1 + (y')^2 = 1 + \left( \sqrt{x} - \frac{1}{4\sqrt{x}} \right)^2 = 1 + \left( x - \frac{1}{2} + \frac{1}{16x} \right) = x + \frac{1}{2} + \frac{1}{16x} = \left( \sqrt{x} + \frac{1}{4\sqrt{x}} \right)^2.$$

Because  $\sqrt{x} + \frac{1}{4\sqrt{x}} \geq 0$ , the surface area is

$$\begin{aligned} 2\pi \int_a^b y\sqrt{1 + (y')^2} \, dx &= 2\pi \int_1^2 \left( \frac{2}{3}x^{3/2} - \frac{\sqrt{x}}{2} \right) \left( \sqrt{x} + \frac{1}{4\sqrt{x}} \right) \, dx \\ &= 2\pi \int_1^2 \left( \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{2}x - \frac{1}{8} \right) \, dx = 2\pi \left( \frac{2x^3}{9} - \frac{x^2}{6} - \frac{1}{8}x \right) \Big|_1^2 = \frac{67}{36}\pi. \end{aligned}$$

11. Compute the total surface area of the coin obtained by rotating the region in Figure 1 about the  $x$ -axis. The top and bottom parts of the region are semicircles with a radius of 1 mm.

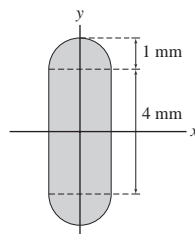


FIGURE 1

This is the equation of the ellipse obtained by translating the ellipse in standard position  $\left(\frac{x}{\frac{5}{2}}\right)^2 + y^2 = 1$  one unit to the right and  $\frac{1}{5}$  unit upward. Since  $a = \frac{5}{2}$ ,  $b = 1$  we have  $c = \sqrt{\left(\frac{5}{2}\right)^2 - 1} \approx 2.3$ , so we obtain the following table:

	Standard position	Translated ellipse
Vertices	$\left(\pm\frac{5}{2}, 0\right), (0, \pm 1)$	$\left(1 \pm \frac{5}{2}, \frac{1}{5}\right), \left(1, \frac{1}{5} \pm 1\right)$
Foci	$(-2.3, 0), (2.3, 0)$	$\left(-1.3, \frac{1}{5}\right), \left(3.3, \frac{1}{5}\right)$
Focal axis	The $x$ -axis	$y = \frac{1}{5}$
Conjugate axis	The $y$ -axis	$x = 1$
Center	The origin	$\left(1, \frac{1}{5}\right)$

In Exercises 39–42, use the Discriminant Test to determine the type of the conic section (in each case, the equation is nondegenerate). Plot the curve if you have a computer algebra system.

39.  $4x^2 + 5xy + 7y^2 = 24$

**SOLUTION** Here,  $D = 25 - 4 \cdot 4 \cdot 7 = -87$ , so the conic section is an ellipse.

41.  $2x^2 - 8xy + 3y^2 - 4 = 0$

**SOLUTION** Here,  $D = 64 - 4 \cdot 2 \cdot 3 = 40$ , giving us a hyperbola.

43. Show that the “conic”  $x^2 + 3y^2 - 6x + 12y + 23 = 0$  has no points.

**SOLUTION** Complete the square in each variable separately:

$$-23 = x^2 - 6x + 3y^2 + 12y = (x^2 - 6x + 9) + (3y^2 + 12y + 12) - 9 - 12 = (x - 3)^2 + 3(y + 2)^2 - 21$$

Collecting constants and reversing sides gives

$$(x - 3)^2 + 3(y + 2)^2 = -2$$

which has no solutions since the left-hand side is a sum of two squares so is always nonnegative.

45. Show that  $\frac{b}{a} = \sqrt{1 - e^2}$  for a standard ellipse of eccentricity  $e$ .

**SOLUTION** By the definition of eccentricity:

$$e = \frac{c}{a} \tag{1}$$

For the ellipse in standard position,  $c = \sqrt{a^2 - b^2}$ . Substituting into (1) and simplifying yields

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

We square the two sides and solve for  $\frac{b}{a}$ :

$$e^2 = 1 - \left(\frac{b}{a}\right)^2 \Rightarrow \left(\frac{b}{a}\right)^2 = 1 - e^2 \Rightarrow \frac{b}{a} = \sqrt{1 - e^2}$$

47. Explain why the dots in Figure 23 lie on a parabola. Where are the focus and directrix located?

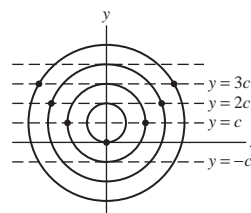


FIGURE 23