

SOLUTIONS MANUAL
to accompany
ROCKET PROPULSION ELEMENTS, 9th EDITION

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This manual is in part an outgrowth of courses taught by Prof. Biblarz, both at the senior/graduate university level and as short courses. All solutions in this manual were prepared by both authors with some student collaboration. A number of design-type problems are included in several of the book's chapters and solutions to these were largely prepared by George Sutton. The style in this manual is informal.

The solutions given are mostly complete but not all problems are included, particularly in the more applied chapters. For problems which are of a "design nature" more than one answer is possible and expected. A few other problems, as presently stated, need additional information or call for assumptions or value-estimates to get started, otherwise the answers can only be given in parametric form. For the most part, all tabular and unit-conversion information may be found in the book but not necessarily in the same chapter as the problem. There are also several problems that require fundamentals not found explicitly in our book – here other standard references in the field should be consulted before searching for specialty papers from the research literature. In some chapters, this manual contains problems not found in the book which been added (together with their solutions) for the instructor's benefit and are labelled "**Extra Problems**".

We have attempted to be clear and accurate in the preparation of this manual. Should you discover errors or deficiencies, or wish to make technical comments about specific solutions, please contact Oscar Biblarz by e-mail at: obiblarz@nps.edu or info@oscarbiblarz.com or by regular mail at: Code ME/Bi, Department of Mechanical & Aerospace Engineering, Naval Postgraduate School, Monterey, CA 93943-5146. Any contributions to problems (particularly those presently without a solution) would be most welcome and may appear in subsequent versions of this manual.

It is intended that the distribution of this Solutions Manual for the 9th edition of Rocket Propulsion Elements be limited to professors, instructors, and other qualified teachers who use our book as a text in college courses. It was prepared to be used by individuals knowledgeable in the field and is not meant for student or other interested reader use. Inquiries for obtaining a copy of this manual should go through an appropriate Wiley sales representative. Visit www.wiley.com for more information.

$$I_s = I_t/w = 10350000/42650 = 237.1 \text{ sec}$$

(b) To determine optimum expansion one needs information from Chapter 3, Fig. 3-4. If $k = 1.25$ and with the given chamber pressure (780 psia) and area ratio (10), at optimum expansion, $p_2 = p_3 = 780/90 = 8.666 \text{ psia}$. The nozzle exit area is $A_2 = 10A_t = 1642 \text{ in}^2$.

From Eq. 2-13 and the given thrust at sea level we can solve for the *momentum thrust*,

$$\dot{m} v_2 = 207,000 - (8.666 - 14.696) = 216,900 \text{ lbf} = \text{thrust at optimum expansion.}$$

$$I_s = 216900/873 = 248.45; \text{ From Appendix 2, the 8.87 is at approximately 4,200 meters.}$$

(c) For vacuum conditions, $p_3 = 0$ and Eq. 2-14 applies,

$$F = 216,900 + 8.67 \times 1642 = 231,000 \text{ lbf} \quad \text{and} \quad I_s = 231000/873 = 295 \text{ sec.}$$

8. During the boost phase of the Atlas V, the RD-180 engine operates together with three solid propellant rocket motors (SRBs) for the initial stage. For the remaining thrust time, the RD-180 operates alone. Using the information given in Table 1-3, calculate the *overall effective exhaust velocity* for the vehicle during the initial combined thrust operation.

The individual mass flow rates for each propulsion type may be inferred from Table 1-3.

$$\text{Main engine: } \dot{m} = (3.82 \times 10^3 \text{ N}) / (337.6 \text{ sec}) \times (9.81 \text{ m/sec}^2) = 1.15 \text{ kg/sec}$$

$$\text{SRBs: } \dot{m} = 3 \times 1.878 \times 10^3 / 279.3 \times 9.81 = 1.965 \text{ kg/sec}$$

Using Equation 2-25, we find

$$(I_s)_{\text{oa}} = \frac{\Sigma F}{g_0 \Sigma \dot{m}} = \frac{(3.82 + 3 \times 1.878) \times 10^3}{9.81 \times (1.15 + 1.965)} = 309 \text{ sec.}$$

9. Using the values given in Table 2-1 for the various propulsion systems, calculate the total impulse for a fixed propellant mass of 20.0 kg.

Use upper values from Table 2-1, and take $m_p = 20.0 \text{ kg}$.

$$I_t = (m_p g_0) I_s = 196.2 I_s \text{ N-sec}$$

Type	I_s (sec)	I_t (N-sec)
Chemical		
Solid	200	3.92×10^4
Liquid	410	8.04×10^4
Monopropellant	223	4.38×10^4
Nuclear Fission	860	1.17×10^5
Resistojet	300	5.89×10^4
Arcjet-thermal	1200	2.35×10^5

$$c^* = 1500 \text{ m/sec}, D_t = 20 \text{ cm}, C_F = 1.38, \dot{m} = 40 \text{ kg/sec}$$

$$c^* = p_1 A_t / \dot{m} \quad \text{so} \quad p_1 \approx p_0 = (1500) \times (40) / (\pi/4) \times (0.20)^2 = 1.91 \text{ MPa}$$

$$F = C_F A_t p_1 \quad \text{so} \quad F = (1.38)(\pi/4) \times (0.20)^2 (2.36 \times 10^6) = 8.28 \times 10^4 \text{ N}$$

$$I_s = F / \dot{m} g_0 \quad \text{so} \quad I_s = (8.29 \times 10^4) / (40)(9.81) = 211 \text{ sec}$$

9. For the rocket unit given in Example 3-2 compute the exhaust velocity if the nozzle is cut off and the exit area is arbitrarily decreased by 50%. Estimate the losses in kinetic energy and thrust and express them as a percentage of the original kinetic energy and the original thrust. [See Fig. 2-1 for subscript notation.]

The wording “old” = original nozzle and “new” = 50% decrease in exit area. The parameters $p_1 = 2.1 \text{ MPa}$, $T_1 = 2222 \text{ K}$, $k = 1.3$ and $A_t = 28.7 \text{ cm}^2$ remain unchanged by the exit area change. Also, because the flow is choked at the throat, the flow rate \dot{m} will not change. Others will change according to the isentropic flow relations in Ch. 3.

	A_2/A_t	M_2	p_2/p_1	T_2/T_1	$a_2 \text{ (m/sec)}$	$v_2 = M_2 a_2 \text{ (m/sec)}$	$F \text{ (kN)}$
Old	3.3	2.6	0.0481	0.497	704	1827	1.827
New	1.65	1.94	0.145	0.64	799	1550	1.750

$$\text{Kinetic Energy Change} = \frac{1}{2} \dot{m} (v_2^2_{\text{new}} - v_2^2_{\text{old}}) / \frac{1}{2} \dot{m} v_2^2_{\text{old}} = -7.20\%$$

$$\text{Thrust Change} = (F_{\text{new}} - F_{\text{old}}) / F_{\text{old}} = -4.21\%$$

10. What is the maximum velocity if the nozzle in Example 3-2 were designed to expand into a vacuum? If the expansion area ratio was 2000?

a) For vacuum with $k = 1.3$, $v_{2\text{max}} = \text{Eqn. 3-18} = 2,580 \text{ m/sec}$

b) With $A_2/A_t = 2000$, from Eqn. 3-14, $M_2 = 9.698$ and $T_2/T_1 = 0.0665$

$$v_2 = M_2 a_2 = 2490 \quad \text{or} \quad 3.5\% \text{ less than } v_{2\text{max}}$$

11. Construction of a variable-area conventional axisymmetric nozzle has often been considered to make the operation of a rocket thrust chamber take place at the optimum expansion ratio at any altitude. Because of the enormous design difficulties of such a mechanical device, it has never been successfully realized. Assuming that such a mechanism can eventually be constructed, what would have to be the variation of the area ratio with altitude (plot up to 50 km) if such a rocket had a chamber pressure of 20 atm? Assume that $k = 1.20$. [The plot below is up to 30 km for greater clarity]

Using Eq. 3-25 at location 2, where $p_2 = p_3$ and the latter is found in Appendix 2,

$$p A_c f = p A_p g \quad \text{and} \quad f + g = 1.50 \text{ m}$$

The resulting moment arms are $f = 0.8608 \text{ m}$ and $g = 0.6496 \text{ m}$. This identifies the location of the center of pressure. The total single force F on the satellite due to solar radiation pressure p is listed next. It is located at the center of pressure.

$$F = p (A_c + A_p) = 1.4267 \times 10^{-6} \times 1.385 = 1.9764 \times 10^{-6} \text{ N}$$

The solar radiation pressure also causes a small linear force displacing the satellite from its orbit, but its effect is of no direct concern to this problem or its solution.

Problem 10 continued, part (a); Torque caused by solar pressure and daily angular displacement.

An unbalanced solar pressure distribution will cause the spacecraft to turn about an axis, which goes through the center of gravity (CG) and is parallel to the cylinder axis. The torque turning the satellite is this force caused by solar radiation pressure (considered to be concentrated at the center of pressure) multiplied by the moment arm e , which is the distance between the center of gravity (CG) and the center of pressure. Here

$$e = f - a = 0.6496 - 0.0554 = 0.5941 \text{ m.}$$

The torque T can now be found

$$T = F e = 1.9764 \times 10^{-6} \times 0.5941 = 1.1741 \times 10^{-6} \text{ Nm}$$

This torque causes an angular acceleration α expressed in radians/sec², which can be determined from $T = I \alpha$ and I is the moment of inertia of the satellite. Solve for α

$$\alpha = T/I = (1.174 \times 10^{-6})/288.8 = 0.44246 \times 10^{-9} \text{ radians/sec}^2$$

The angle through which the satellite is turned is θ ($\theta = \frac{1}{2} \alpha t^2$) and the time t here is one day or 24 hours or 86,400 seconds. However this time is reduced to 54 %, because the satellite is in the shadow of the planet earth (no radiation received) during parts of its flight. So the effective time is $t = 86,400 \text{ sec} \times 0.540 = 46,656 \text{ sec.}$

The angle traveled in one day is

$$\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \times 0.40654 \times 10^{-9} \times 46,656^2 = 0.4424 \text{ radians per day or about } 25^\circ/\text{day.}$$

The attitude control system explained in part (c) is intended to remedy this angle change.

Problem 10, continued, part (b); calculate the drag force and compare it with solar radiation forces. Since an accurate determination of the variable drag is beyond the scope of this book, an order of magnitude simple solution will be considered to be satisfactory.

(b) Eq. 3-4 in terms of the density, $\rho = p/RT = 21.2 \text{ kg/m}^3$ so $\text{Vol})_{\text{H}_2} = 696/21.2 = 32.83 \text{ m}^3$

(c) $\text{Vol})_{\text{HTP}} = 593/1.46 \times 10^3 = 0.406 \text{ m}^3$

(d) Because 2 moles of HTP are needed to produce each mol of $\text{O}_2(\text{g})$ compared to one mol of $\text{O}_2(\text{g})$ from an oxygen tank, we may define a *mass overhead* as the ratio of the molar masses of HTP to O_2 in the decomposition equation above: $\text{mass overhead} = 2 \times 34.016/31.999 = 2.126$.

thrust is 10^{-5} N per accelerator hole, that there are several thousand holes of “*aspect*

ratio” $D/d = 1.0$ in the accelerator, and that the neutralizer operates with protons

(which have a mass 1836 times that of the electron).

a) $V_{acc} = [F/6.18 \times 10^{-12}]^{1/2} = 1.27 \times 10^3$ Volts (Eq. 17-24 with $D/d = 1.0$)

b) Electron current per hole:

$$I = Aj = \left(\frac{\pi}{4} D^2 \right) \left(\frac{4\epsilon_0}{9} \sqrt{\frac{2e}{\mu}} \frac{V_{acc}^{3/2}}{d^2} \right) = \frac{8.85 \times 10^{-12} \pi}{9} \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} (1.27 \times 10^3)^{3/2} \left(\frac{D}{d} \right)^2$$

$$I = 8.31 \times 10^{-3} \text{ Amps/hole (which is a very substantial amount of current).}$$

c) Conservation of linear momentum in neutralizer region (idealized case):

$$\mu v_{electron} = (\mu + m_{proton}) v_{mix} \text{ or } v_{mix} \approx v_{electron}/1822$$

$$v_{electron} = \sqrt{2eV_{acc} / \mu} = 2.11 \times 10^7 \text{ m/sec (non-relativistic)}$$

$$v_{mix} \approx 2.11 \times 10^7 / 1822 = 1.16 \times 10^4 \text{ m/sec or } I_s = 1,182 \text{ sec (exit beam value)}$$

d) Vast amounts of momentum have been lost in accelerating the protons, and beam losses during mixing would likely be catastrophic. Also, sourcing such a large proton neutralizing current will be quite difficult in practice. So perhaps this inventor should give consideration to neutralizing with *positrons* (pending several technological breakthroughs). All of this should be revisited for the case when more energetic/massive relativistic electrons are present.

12. For each of the three thrusters in Example 17-4, calculate the thrust F and input power P_e that would apply for a payload mass m_{pl} of 100 kg. What would result if the spacecraft power supply is limited to 30 kW but mission time could extend up to 100 days?

Using Eqs. 17-3 to 17-6 we can arrive at the following,

$$F = \frac{m_0 - m_{pl}}{\frac{t_p}{g_0 I_s} + \frac{g_0 I_s}{2\alpha \eta_t}} \quad \text{and} \quad P = \frac{g_0 I_s}{2\eta_t} F$$

	I_s (sec)	α (W/kg)	η_t	t_p (sec)	m_0 (kg)	F (N)	P_e (kW)
Arcjet	800	295	0.32	1.296×10^6	255.8	0.754	9.24
Hall	1983	366	0.57	0.588×10^6	233.1	1.73	29.6
Ion	2800	278	0.50	0.864×10^6	294.1	1.49	40.9