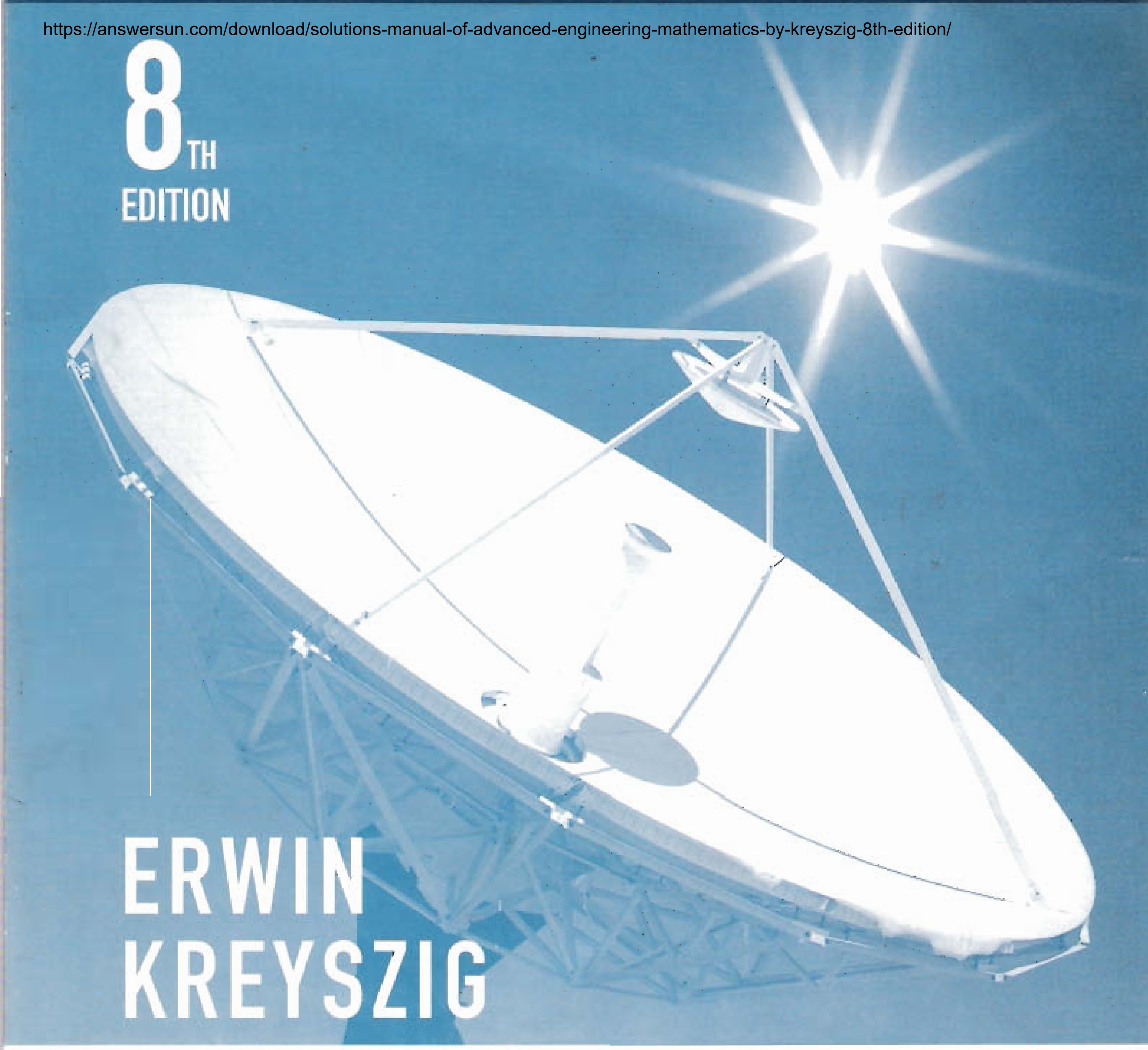
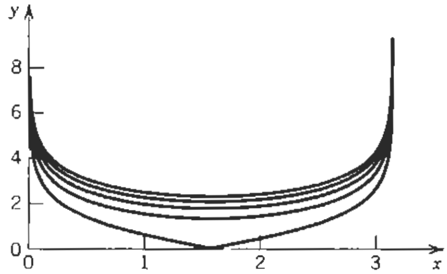
8TH EDITION



ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

INSTRUCTOR'S MANUAL



Section 1.5. Problem 6

8. $y/x = c$

10. $re^{3\theta} = c$

12. $x \cot y + x^3/3 = c$

14. Yes, $y = 3.8 \sin 2x$

16. Yes, $y^2 + ye^x = 0$

18. $(\omega \cos \omega y)_x = 0$ but $(2 \sin \omega y)_y \neq 0$. Not exact. By separation of variables,

$$\cot \omega y \, dy = -\frac{2}{\omega} dx, \qquad \frac{1}{\omega} \ln |\sin \omega y| = -\frac{2}{\omega} x + \tilde{c}, \qquad \sin \omega y = ce^{-2x}.$$

Now $y(0) = \pi/(2\omega)$ gives $\sin(\pi/2) = c$. Hence $c = 1$. *Answer:* $e^{2x} \sin \omega y = 1$.

20. $(2xye^{x^2})_y = 2xe^{x^2} = (e^{x^2})_x$ shows exactness. By integration,

$$ye^{x^2} = c.$$

$y(0) = 2$ gives $c = 2$. *Answer:* $y = 2e^{-x^2}$.

22. Equation (9) becomes $s^4 + t^4 = const$; see the figure in the solution to Prob. 12 of Sec. 1.1 in this Manual.

24. $(xy)^{-1} dy - x^2 dx = 0$ has the integrating factor $F = y$, giving

$$\text{(A)} \qquad x^{-1} dy - x^{-2} y \, dx = 0,$$

which is exact because

$$(x^{-1})_x = -x^{-2} = (-x^{-2}y)_y.$$

Now (A) implies

$$x^{-1} dy - x^{-2} y \, dx = \frac{x \, dy - y \, dx}{x^2} = d\left(\frac{y}{x}\right) = 0$$

so that

$$\frac{y}{x} = c, \qquad y = cx$$

as claimed, but $y \equiv 0$ is not a solution of the original equation.

26. $y \cos(x + y) \, dx + [y \cos(x + y) + \sin(x + y)] \, dy = 0$ is exact because

$$\begin{aligned}[y \cos(x + y)]_y &= \cos(x + y) - y \sin(x + y) \\ &= [y \cos(x + y) + \sin(x + y)]_x.\end{aligned}$$

By inspection or systematically,

$$y \sin(x + y) = c.$$

From this and (7),

$$y_p = e^{2x}\frac{1}{7}x^{7/2} - xe^{2x}\frac{2}{5}x^{5/2} + x^2e^{2x}\frac{1}{3}x^{3/2}$$
$$= e^{2x}x^{7/2}\frac{1}{105}(15 - 42 + 35) = \frac{8}{105}e^{2x}x^{7/2}.$$

Answer:

$$y = \left[c_1 + c_2x + c_3x^2 + \tfrac{8}{105}x^{7/2}\right]e^{2x}.$$

8. $y_1 = x$, $y_2 = x^{1/2}$, $y_3 = x^{3/2}$, $W = -\frac{1}{4}$, $W_1 = x$, $W_2 = -\frac{1}{2}x^{3/2}$, $W_3 = -\frac{1}{2}x^{1/2}$, $r = x^{5/2}$ (divide by $4x^3$). From (7) we thus obtain

$$y_p = x\int(-4x)x^{5/2}\,dx + x^{1/2}\int 2x^{3/2}x^{5/2}\,dx + x^{3/2}\int 2x^{1/2}x^{5/2}\,dx$$
$$= -\tfrac{8}{9}x^{11/2} + \tfrac{2}{5}x^{11/2} + \tfrac{1}{2}x^{11/2}$$
$$= \tfrac{1}{90}x^{11/2}.$$

Answer:

$$y = c_1x + c_2x^{1/2} + c_3x^{3/2} + x^{11/2}/90.$$

10. $y_1 = x$, $y_2 = x\ln x$, $y_3 = x(\ln x)^2$, $W = 2$, $W_1 = x(\ln x)^2$, $W_2 = -2x\ln x$, $W_3 = x^2$, $r = 1/x$. *Answer:* $y = x^2 + x\ln x$

12. $y = \sin x + \sin 3x + 2\sinh x$

14. CAS PROJECT. The first equation has as a general solution

$$y = (c_1 + c_2x + c_3x^2)e^x + \tfrac{8}{105}e^xx^{7/2},$$

so in cases such as this, one could try

$$y = x^{1/2}(a_0 + a_1x + a_2x^2 + a_3x^3)e^x.$$

However, the equation alone does not show much, so another idea is needed. One could modify the right side systematically and see how the solution changes. The solution of the second suggested equation shows that the equation is not accessible by undetermined coefficients; its solution is (see Prob. 2)

$$y = c_1x^{-1} + c_2x + c_3x^2 + \tfrac{1}{8}x^3\ln x - \tfrac{7}{32}x^3.$$

And one could perhaps modify this equation, too, in an attempt to obtain a form of solution that might be suitable for undetermined coefficients.

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16. $y = e^{-3x}(A\cos\frac{1}{2}x + B\sin\frac{1}{2}x)$

18. $y = c_1x^3 + c_2x^{-3}$

20. y_p is obtained by the method of undetermined coefficients. *Answer:*

$$y = c_1e^x + c_2e^{-x} + c_3e^{4x} - 5e^{2x}.$$

22. The particular solution $y_p = x^2e^{\pi x}$ is obtained by the method of undetermined coefficients. *Answer:*

$$y = (c_1 + c_2x)e^{\pi x} + x^2e^{\pi x}.$$

4. $sY_1 + 3 = 5Y_1 + Y_2, \quad sY_2 - 7 = Y_1 + 5Y_2.$ *Answer:*

$$y_1 = 2e^{6t} - 5e^{4t}, \qquad y_2 = 2e^{6t} + 5e^{4t}.$$

6. $y_1 = \sin t + \cos 2t, \quad y_2 = \sin t - \cos 2t$

8. The subsidiary equations are

$$s^2Y_1 - 3s = -5Y_1 + 2Y_2, \qquad s^2Y_2 - s = 2Y_1 - 2Y_2.$$

They have the solutions

$$Y_1 = \frac{3s^3 + 8s}{(s^2+1)(s^2+6)} = \frac{s}{s^2+1} + \frac{2s}{s^2+6}$$

$$Y_2 = \frac{s^3 + 11s}{(s^2+1)(s^2+6)} = \frac{2s}{s^2+1} - \frac{s}{s^2+6}.$$

Answer:

$$y_1 = \cos t + 2\cos\sqrt{6}\,t, \qquad y_2 = 2\cos t - \cos\sqrt{6}\,t.$$

10. $y_1 = t^2, y_2 = t^2 + 2t, y_3 = t^2 - 2t$

12. The subsidiary equations are

$$sY_1 + Y_2 = \frac{2s}{s^2+1}(1 - e^{-2\pi s}), \qquad Y_1 + sY_2 = 1.$$

Solving algebraically gives

$$Y_1 = \frac{1}{s^2+1} - \frac{2s^2e^{-2\pi s}}{s^4 - 1}$$

$$Y_2 = \frac{s}{s^2+1} + \frac{2se^{-2\pi s}}{s^4-1}.$$

Answer:

$$y_1 = \sin t \quad \text{if } 0 \leqq t \leqq 2\pi, \qquad y_1 = -\sinh(t - 2\pi) \quad \text{if } t > 2\pi$$

$$y_2 = \cos t \quad \text{if } 0 \leqq t \leqq 2\pi, \qquad y_2 = \cosh(t - 2\pi) \quad \text{if } t > 2\pi.$$

14. The subsidiary equations are

$$sY_1 + 4 = 64\left(\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}\right) + 2Y_1 + 4Y_2, \qquad sY_2 + 4 = Y_1 + 2Y_2.$$

Solving algebraically gives

$$Y_1 = \frac{-4s - 8}{s(s-4)} - \frac{64(2 + s - s^2)e^{-s}}{s^3(s-4)}$$

$$Y_2 = \frac{-4s + 4}{s(s-4)} + \frac{64(1+s)e^{-s}}{s^3(s-4)}.$$

Taking the inverse Laplace transform gives the *answer*

$$y_1 = -6e^{4t} + 2 + u(t-1)[-18 + 10e^{4t-4} - 8t + 16t^2]$$

$$y_2 = -3e^{4t} - 1 + u(t-1)[7 + 5e^{4t-4} - 4t - 8t^2].$$

16. $y_1 = 100 - 62.5e^{-0.24t} - 37.5e^{-0.08t}$,
$y_2 = 100 + 125e^{-0.24t} - 75e^{-0.08t}$.
Setting $2t = \tau$ gives the old solution, except for notation.

It can be obtained in several ways. (a) Integrate the Maclaurin series of the integrand termwise and form the Cauchy product with the series of e^{z^2}. (b) f satisfies the differential equation $f' = 2zf + 1$. Use this, its derivatives $f'' = 2(f + zf')$, etc., $f(0) = 0$, $f'(0) = 1$, etc., and the coefficient formulas in (1). (c) Substitute $f = \sum_{n=0}^{\infty} a_n z^n$ and $f' = \sum_{n=0}^{\infty} n a_n z^{n-1}$ into the differential equation and compare coefficients; that is, apply the power series method (Sec. 4.1).

10. $z - \frac{z^3}{3!3} + \frac{z^5}{5!5} - \frac{z^7}{7!7} + - \cdots; \quad R = \infty$

12. $z - \frac{z^5}{2!5} + \frac{z^9}{4!9} - \frac{z^{13}}{6!13} + - \cdots; \quad R = \infty$

14. First of all, since $\sin(w + 2\pi) = \sin w$ and $\sin(\pi - w) = \sin w$, we obtain all values of $\sin w$ by letting w vary in a suitable vertical strip of width π, for example, in the strip $-\pi/2 \leqq u \leqq \pi/2$. Now since

$$\sin\left(\frac{\pi}{2} - iy\right) = \sin\left(\frac{\pi}{2} + iy\right) = \cosh y$$

and

$$\sin\left(-\frac{\pi}{2} - iy\right) = \sin\left(-\frac{\pi}{2} + iy\right) = -\cosh y,$$

we have to exclude a part of the boundary of that strip, so we exclude the boundary in the lower half-plane. To solve our problem we have to show that the value of the series lies in that strip. This follows from $|z| < 1$ and

$$\left|\operatorname{Re}\left(z + \frac{1}{2}\frac{z^3}{3} + \cdots\right)\right| \leqq \left|z + \frac{1}{2}\frac{z^3}{3} + \cdots\right| \leqq |z| + \frac{1}{2}\frac{|z|^3}{3} + \cdots$$
$$= \sin^{-1}|z| < \frac{\pi}{2}.$$

16. $\frac{1}{i(1 - i(z - i))} = -i \sum_{n=0}^{\infty} i^n (z - i)^n; \quad R = 1$

18. $[(z + 1) - 1]^5 = -1 + 5(z + 1) - 10(z + 1)^2 + 10(z + 1)^3 - 5(z + 1)^4 + (z + 1)^5$

20. $\cos \pi z = -\sin\left(\pi z - \frac{1}{2}\pi\right) = -\frac{\pi}{1!}\left(z - \frac{1}{2}\right) + \frac{\pi^3}{3!}\left(z - \frac{1}{2}\right)^3 - + \cdots; \quad R = \infty$

22. $1 + \frac{1}{2!}(z - \pi i)^2 + \frac{1}{4!}(z - \pi i)^4 + \frac{1}{6!}(z - \pi i)^6 + \cdots; \quad R = \infty$

24. $-\frac{1}{4} - \frac{2i}{8}(z - i) + \frac{3}{16}(z - i)^2 + \frac{4i}{32}(z - i)^3 - \frac{5}{64}(z - i)^4 + \cdots; \quad R = 2$

26. We obtain

$$\cos^2 z = \frac{1}{2} + \frac{1}{2}\cos 2z = \frac{1}{2} - \frac{1}{2}\cos(2z - \pi)$$
$$= \frac{1}{2}\left[\frac{4}{2!}\left(z - \frac{1}{2}\pi\right)^2 - \frac{4^2}{4!}\left(z - \frac{1}{2}\pi\right)^4 + - \cdots\right]; \quad R = \infty$$

In the further integrations we can use the defining integrals of $E[X - \mu)^2]$ and $E[(X - \mu)^3]$ or, more simply,

$$\sigma^2 = E(X^2) - \mu^2 = 2\int_0^1 x^2(1 - x)\,dx - \frac{1}{9} = 2\left[\frac{1}{3} - \frac{1}{4}\right] - \frac{1}{9} = \frac{1}{18}$$

and similarly,

$$\begin{aligned} E[(X - \mu)^3] &= E(X^3) - 3\mu E(X^2) + 3\mu^2 E(X) - \mu^3 \\ &= E(X^3) - 3\mu E(X^2) + 2\mu^3 \\ &= 2\int_0^1 x^3(1 - x)\,dx - 3\cdot\frac{1}{3}\cdot\frac{1}{6} + \frac{2}{27} \\ &= 2\left[\frac{1}{4} - \frac{1}{5}\right] - \frac{1}{6} + \frac{2}{27} = \frac{1}{135}. \end{aligned}$$

This gives

$$\gamma = \frac{18\sqrt{18}}{5 \cdot 27} = \frac{2\sqrt{2}}{5}.$$

48. 0.1587, 0.6306, 0.5, 0.4950

50. 25.71 cm, 0.0205 cm