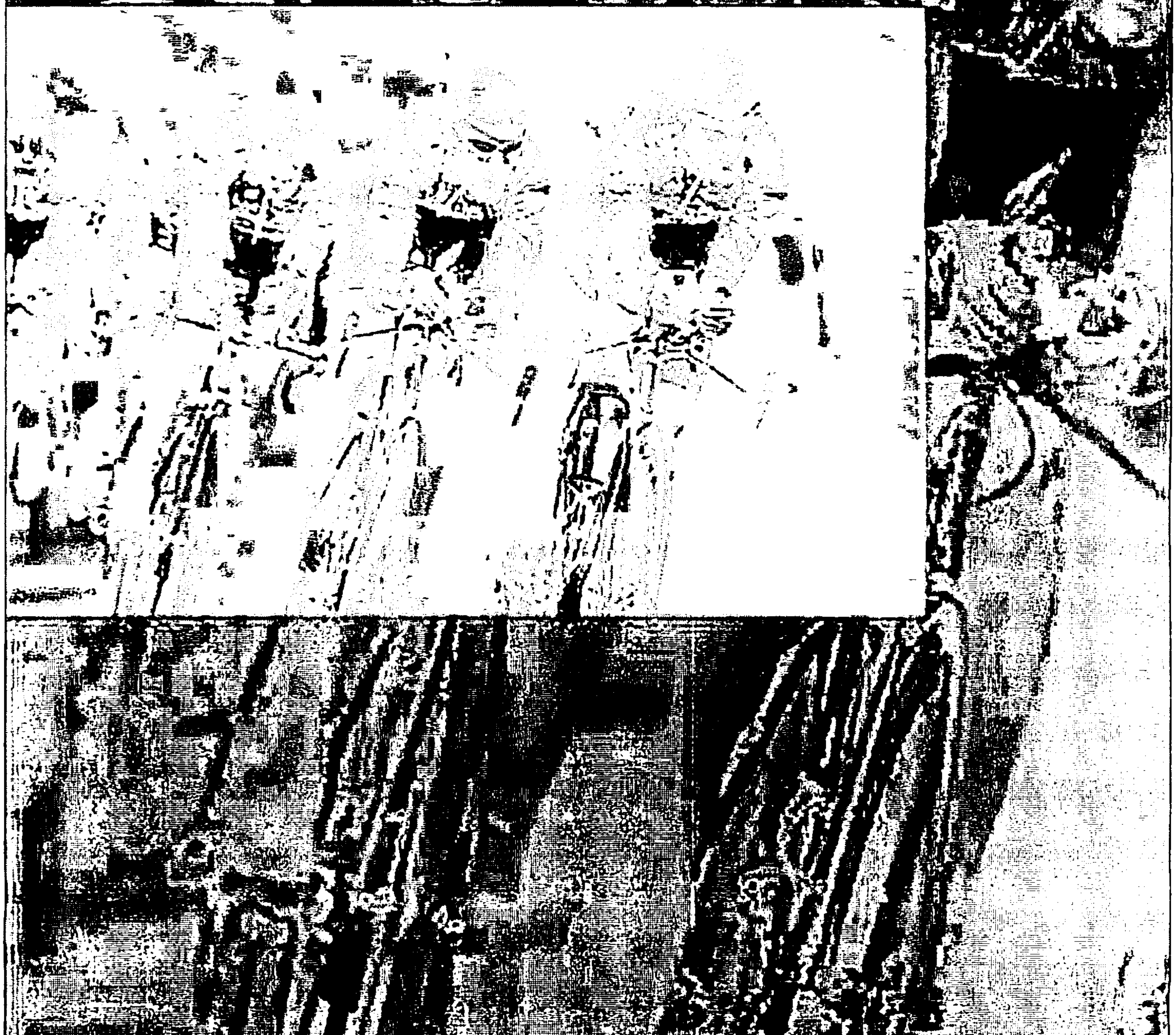
INSTRUCTOR'S SOLUTIONS MANUAL

SEVENTH EDITION

ANALYTICAL MECHANICS



FOWLES & CASSIDAY

1.24 $$\frac{d}{dt}\left[\bar{r}\cdot\left(\bar{v}\times\bar{a}\right)\right]=\frac{d\bar{r}}{dt}\cdot\left(\bar{v}\times\bar{a}\right)+\bar{r}\cdot\frac{d}{dt}\left(\bar{v}\times\bar{a}\right)$$

$$=\bar{v}\cdot\left(\bar{v}\times\bar{a}\right)+\bar{r}\cdot\left[\left(\frac{d\bar{v}}{dt}\times\bar{a}\right)+\left(\bar{v}\times\frac{d\bar{a}}{dt}\right)\right]$$

$$=0+\bar{r}\cdot\left[0+\left(\bar{v}\times\dot{\bar{a}}\right)\right]$$

$$\frac{d}{dt}\left[\bar{r}\cdot\left(\bar{v}\times\bar{a}\right)\right]=\bar{r}\cdot\left(\bar{v}\times\dot{\bar{a}}\right)$$

1.25 $\vec{v}=v\hat{\tau}$ and $\vec{a}=a_\tau\hat{\tau}+a_n\hat{n}$

$\vec{v}\cdot\vec{a}=va_\tau$, so $a_\tau=\dfrac{\vec{v}\cdot\vec{a}}{v}$

$a^2=a_\tau^2+a_n^2$, so $a_n=\left(a^2-a_\tau^2\right)^{\frac{1}{2}}$

1.26 For 1.14, $$a_\tau=\frac{-b^2\omega^3\cos\omega t\cdot\sin\omega t+b^2\omega^3\sin\omega t\cdot\cos\omega t+4c^2t}{\left(b^2\omega^2\cos^2\omega t+b^2\omega^2\sin^2\omega t+4c^2t^2\right)^{\frac{1}{2}}}$$

$$a_\tau=\frac{4c^2t}{\left(b^2\omega^2+4c^2t^2\right)^{\frac{1}{2}}}$$

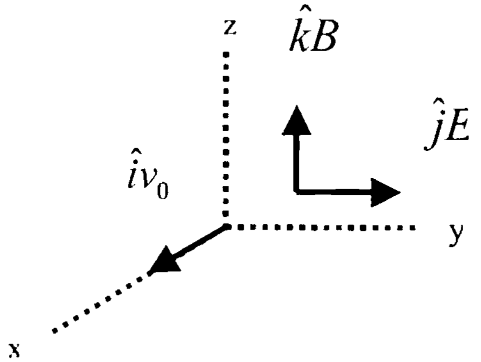
$$a_n=\left(b^2\omega^2+4c^2-\frac{16c^4t^2}{b^2\omega^2+4c^2t^2}\right)^{\frac{1}{2}}$$

For 1.15, $$a_\tau=\frac{b^2k\left(k^2-c^2\right)e^{2kt}+2b^2c^2ke^{2kt}}{be^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}}=bke^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}$$

$$a_n=\left[b^2e^{2kt}\left(k^2+c^2\right)^2-b^2k^2e^{2kt}\left(k^2+c^2\right)\right]^{\frac{1}{2}}=bce^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}$$

1.27 $\vec{v}=v\hat{\tau}$, $\vec{a}=\dot{v}\hat{\tau}+\dfrac{v^2}{\rho}\hat{n}$

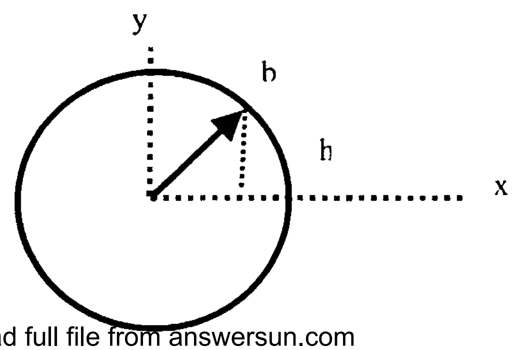
$$\left|\vec{v}\times\vec{a}\right|=v\cdot a_n=v\frac{v^2}{\rho}=\frac{v^3}{\rho}$$



$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$
$$\vec{v}\times\vec{B} = \left(\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}\right)\times\hat{k}B = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$
$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q\left(E - \dot{x}B\right)$$
$$m\ddot{x} = F_x = q\dot{y}B$$
$$\dot{x} - \dot{x}_\circ = \frac{qB}{m}y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_\circ + \frac{qB}{m}y\right)$$
$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}_\circ}{m} - \left(\frac{qB}{m}\right)^2 y = -\frac{eE}{m} + \frac{eB\dot{x}_\circ}{m} - \left(\frac{eB}{m}\right)^2 y$$
$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_\circ, \qquad \omega = \frac{eB}{m}$$
$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega x_\circ\right) + A\cos\left(\omega t + \theta_\circ\right)$$
$$\dot{y} = -A\omega\sin\left(\omega t + \theta_\circ\right)$$
$$\dot{y}_\circ = 0, \text{ so } \quad \theta_\circ = 0$$
$$y_\circ = 0, \text{ so } \quad A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_\circ\right)$$
$$y = a\left(1 - \cos\omega t\right), \qquad a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_\circ\right)$$
$$\dot{x} = \dot{x}_\circ + \frac{qB}{m}y = \dot{x}_\circ - \omega y = \dot{x}_\circ - \omega a\left(1 - \cos\omega t\right)$$
$$\dot{x} = \left(\dot{x}_\circ - \omega a\right) + \omega a\cos\omega t$$
$$x = \left(\dot{x}_\circ - \omega a\right)t + a\sin\omega t$$
$$x = a\sin\omega t + bt, \qquad b = \dot{x}_\circ - \omega a$$
$$m\ddot{z} = F_z = 0$$
$$z = \dot{z}_\circ t + z_\circ = 0$$

4.21



$$\frac{1}{2}mv^2 + mqh = mg\frac{b}{2}$$
$$v^2 = g\left(b - 2h\right)$$
$$F_r = -\frac{mv^2}{b} = -mg\cos\theta + R$$

For $r = a\theta$, and the force to be central, try $\theta = bt^n$

$$f(\theta) = m\left[2ab^2n^2t^{2n-2} + ab^2n(n-1)t^{2n-2}\right]$$

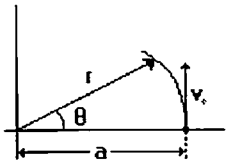
For a central field … $f(\theta) = 0$

$$2n + (n-1) = 0$$

$$n = \frac{1}{3}$$

$$\theta = bt^{\frac{1}{3}}$$

6.14



(a)

Calculating the potential energy

$$-\frac{dv}{dr} = f(r) = -k\left(\frac{4}{r^3} + \frac{a^2}{r^5}\right)$$

Thus, $V = -k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right)$

The total energy is …

$$E = T_o + V_o = \frac{1}{2}v_o^2 - k\left(\frac{2}{a^2} + \frac{1}{4a^2}\right) = \frac{1}{2}\left(\frac{9k}{2a^2}\right) - \frac{9k}{4a^2} = 0$$

Its angular momentum is …

$$l^2 = a^2v_o^2 = \frac{9k}{2} = constant = r^4\dot{\theta}^2$$

Its KE is …

$$T = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\dot{\theta}^2 = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{l^2}{r^4}$$

The energy equation of the orbit is …

$$T + V = 0 = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{l^2}{r^4} - k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right)$$

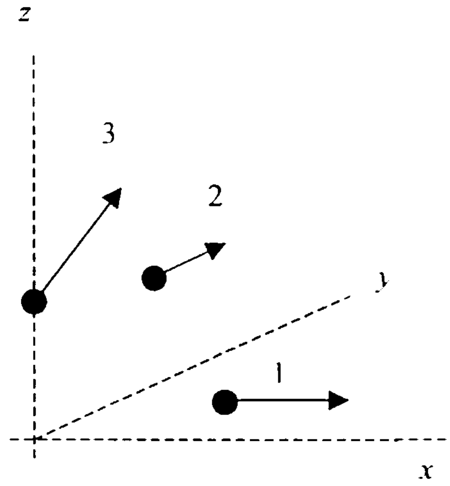
$$= \left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{9k}{4r^4} - k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right)$$

or $\left(\frac{dr}{d\theta}\right)^2 = \frac{1}{9}\left(a^2 - r^2\right)$

Letting $r = a\cos\phi$ then $\frac{dr}{d\theta} = -a\sin\phi\frac{d\phi}{d\theta}$

So $\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{9}$ $\therefore \phi = \frac{1}{3}\theta$

Chapter 7
Dynamics of Systems of Particles



7.1 From eqn. 7.1.1, $\vec{r}_{cm} = \frac{1}{m}\sum_i m_i\vec{r}_i$

$$\vec{r}_{cm} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) = \frac{1}{3}(\hat{i} + \hat{j} + \hat{j} + \hat{k} + \hat{k})$$

$$\vec{r}_{cm} = \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{v}_{cm} = \frac{d}{dt}\vec{r}_{cm} = \frac{1}{3}(\vec{v}_1 + \vec{v}_2 + \vec{v}_3) = \frac{1}{3}(2\hat{i} + \hat{j} + \hat{i} + \hat{j} + \hat{k})$$

$$\vec{v}_{cm} = \frac{1}{3}(3\hat{i} + 2\hat{j} + \hat{k})$$

From eqn 7.1.3, $\vec{p} = \sum_i m_i\vec{v}_i = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$

$$\vec{p} = 3\hat{i} + 2\hat{j} + \hat{k}$$

7.2 (a) From eqn. 7.2.15, $T = \sum_i \frac{1}{2} m_i v_i^2$

$$T = \frac{1}{2}\left[2^2 + 1^2 + (1^2 + 1^2 + 1^2)\right] = 4$$

(b) From Prob. 7.1, $\vec{v}_{cm} = \frac{1}{3}(3\hat{i} + 2\hat{j} + \hat{k})$

$$\frac{1}{2}mv_{cm}^2 = \frac{1}{2}\times 3\times\frac{1}{9}(3^2 + 2^2 + 1^2) = 2\frac{1}{3}$$

(c) From eqn.7.2.8, $\vec{L} = \sum_i \vec{r}_i \times m\vec{v}_i$

$$\vec{L} = \left[(\hat{i} + \hat{j})\times 2\hat{i}\right] + \left[(\hat{j} + \hat{k})\times\hat{j}\right] + \left[\hat{k}\times(\hat{i} + \hat{j} + \hat{k})\right]$$

$$\vec{L} = (-2\hat{k}) + (-\hat{i}) + (\hat{j} - \hat{i}) = -2\hat{i} + \hat{j} - 2\hat{k}$$

7.3 $\vec{v}_\circ = \vec{v}_b - \vec{v}_g$

Since momentum is conserved and the bullet and gun were initially at rest:

$$0 = m\vec{v}_b + M\vec{v}_g$$

$$\vec{v}_g = -\gamma\vec{v}_b, \qquad \gamma = \frac{m}{M}$$

$$\vec{v}_\circ = (1+\gamma)\vec{v}_b$$

$$\vec{v}_b = \frac{\vec{v}_\circ}{1+\gamma}$$

$$= C - \frac{1}{2}gt^2 - \frac{gt}{k}\ln(1-kt) - \frac{g}{k}\int\left(-1 + \frac{1}{1-kt}\right)dt$$

$$= C - \frac{1}{2}gt^2 - \frac{gt}{k}\ln(1-kt) + \frac{gt}{k} + \frac{g}{k^2}\ln(1-kt)$$

$$= C - \frac{1}{2}gt^2 + \frac{g}{k^2}(1-kt)\ln(1-kt) + \frac{gt}{k}$$

but $y = 0$ at $t = 0$ so $C = 0$

$$y = \frac{gt}{k} - \frac{1}{2}gt^2 + \frac{g}{k^2}(1-kt)\ln(1-kt)$$

and at $t = t_\circ$

(a) $$H = \frac{gt_\circ^2}{2m_\circ^2}\left[(2M + m_\circ)m_\circ + 2M(M + m_\circ)\ln\left(\frac{M}{M + m_\circ}\right)\right]$$

(b) $$v = \frac{gt_\circ}{m_\circ}\left[(M + m_\circ)\ln\frac{(M + m_\circ)}{M} - m_\circ\right]$$

(c) letting $\varepsilon = \frac{m_\circ}{M} << 1$ we have

$$H = \frac{gt_\circ^2}{2\varepsilon^2}\left[(2+\varepsilon)\varepsilon - 2(1+\varepsilon)\ln(1+\varepsilon)\right]$$

$$= \frac{gt_\circ^2}{2\varepsilon^2}\left[2\varepsilon + \varepsilon^2 - 2(1+\varepsilon)\left(\varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \ldots\right)\right]$$

$$H \cong \frac{gt_\circ^2}{6}\varepsilon$$

Similarly:

$$v = \frac{gt_\circ}{\varepsilon}\left[(1+\varepsilon)\ln(1+\varepsilon) - \varepsilon\right]$$

$$= \frac{gt_\circ}{\varepsilon}\left[(1+\varepsilon)\left(\varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \ldots\right) - \varepsilon\right]$$

$$\approx \frac{1}{2}gt_\circ\varepsilon$$

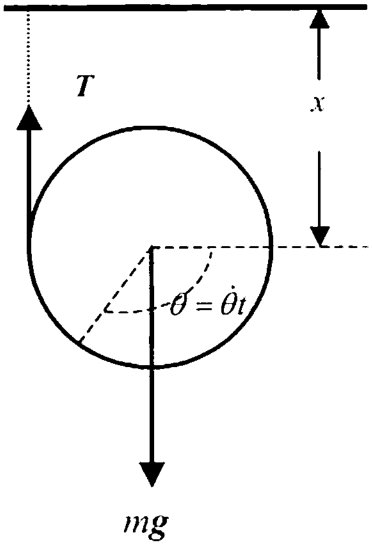
(d) $H = 327m$: $v = 9.8ms^{-1}$

7.26 $\dot{m} = -k$ or $m = m_\circ - kt$

Burn-out occurs at time $T = \frac{\varepsilon m_\circ}{k}$

So – the rocket equation (7.7.7) becomes

$$m\frac{dv}{dt} = -V\dot{m}$$ (-) since V is oppositely directed to $\dot{v}$



$$L = T - V = \frac{1}{2}m\dot{x}^2 + \frac{1}{5}ma^2\dot{\theta}^2 + mgx$$

The equation of constraint is …

$$f(x,\theta) = x - a\theta = 0$$

The 2 Lagrange equations with multipliers are …

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \lambda\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} + \lambda\frac{\partial f}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x} \qquad \frac{\partial L}{\partial x} = mg \qquad \lambda\frac{\partial f}{\partial x} = \lambda$$

$mg - m\ddot{x} + \lambda = 0$ and from the θ-equation …

$$-\frac{2}{5}ma^2\ddot{\theta} - \lambda a = 0$$

Differentiating the equation of constraint … $\ddot{\theta} = \frac{\ddot{x}}{a}$ and substituting into the above …

$$\ddot{x} = \frac{5}{7}g \quad \text{and} \quad \lambda = -\frac{2}{7}mg$$

(b) The generalized force that is equivalent to the tension T is …

$$Q_x = \lambda\frac{\partial f}{\partial x} = \lambda = -\frac{2}{7}mg$$

10.16 For $\vec{v}'$ the velocity of a differential mass element, dm, of the spring at a distance x' below the support …

$$\vec{v}' = \frac{x'}{x}\vec{v}, \qquad dm = \frac{m'}{x}dx'$$

$$T = \frac{1}{2}mv^2 + \int_0^{m'}\frac{1}{2}(v')^2\,dm = \frac{1}{2}m\dot{x}^2 + \int_0^{x}\frac{1}{2}\left(\frac{x'}{x}\dot{x}\right)^2\frac{m'}{x}dx'$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\frac{m'}{3}\dot{x}^2$$

$$V = \frac{1}{2}k(x-l)^2 - mgx - \int_0^{m'} gx'dm'$$

$$= \frac{1}{2}k(x-l)^2 - mgx - \int_0^{x} gx'\frac{m'}{x}dx'$$

$$V = \frac{1}{2}k(x-l)^2 - mgx - \frac{m'}{2}gx$$

$$L = T - V = \frac{1}{2}\left(m + \frac{m'}{3}\right)\dot{x}^2 - \frac{1}{2}k(x-l)^2 + \left(m + \frac{m'}{2}\right)gx$$

$$\frac{\partial L}{\partial \dot{x}} = \left(m + \frac{m'}{3}\right)\dot{x}, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \left(m + \frac{m'}{3}\right)\ddot{x},$$