CHAPTER 0 Preliminaries

0.1 Concepts Review

1. rational numbers
2. dense
3. If not Q then not P.
4. theorems

Problem Set 0.1

1. $4-2(8-11)+6=4-2(-3)+6$
$=4+6+6=16$

2. $3\left[2-4(7-12)\right]=3\left[2-4(-5)\right]$
$=3\left[2+20\right]=3(22)=66$

3. $-4[5(-3+12-4)+2(13-7)]$
$=-4[5(5)+2(6)]=-4[25+12]$
$=-4(37)=-148$

4. $5\left[-1(7+12-16)+4\right]+2$
$=5\left[-1(3)+4\right]+2=5(-3+4)+2$
$=5(1)+2=5+2=7$

5. $\frac{5}{7}-\frac{1}{13}=\frac{65}{91}-\frac{7}{91}=\frac{58}{91}$

6. $\frac{3}{4-7}+\frac{3}{21}-\frac{1}{6}=\frac{3}{-3}+\frac{3}{21}-\frac{1}{6}$
$=-\frac{42}{42}+\frac{6}{42}-\frac{7}{42}=-\frac{43}{42}$

7. $\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{3}\right)+\frac{1}{6}\right]=\frac{1}{3}\left[\frac{1}{2}\left(\frac{3-4}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[\frac{1}{2}\left(-\frac{1}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[-\frac{1}{24}+\frac{4}{24}\right]$
$=\frac{1}{3}\left(\frac{3}{24}\right)=\frac{1}{24}$

8. $-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)\right]=-\frac{1}{3\left[\frac{2}{5}-\frac{1}{2}\left(\frac{5}{15}-\frac{3}{15}\right)\right]}$
$=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{2}{15}\right)\right]=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{15}\right]$
$=-\frac{1}{3}\left(\frac{6}{15}-\frac{1}{15}\right)=-\frac{1}{3}\left(\frac{5}{15}\right)=-\frac{1}{9}$

9. $\frac{14}{21}\left(\frac{2}{5-\frac{1}{3}}\right)^2=\frac{14}{21}\left(\frac{2}{\frac{14}{3}}\right)^2=\frac{14}{21}\left(\frac{6}{14}\right)^2$
$=\frac{14}{21}\left(\frac{3}{7}\right)^2=\frac{2}{3}\left(\frac{9}{49}\right)=\frac{6}{49}$

10. $\frac{\left(\frac{2}{7}-5\right)}{\left(1-\frac{1}{7}\right)}=\frac{\left(\frac{2}{7}-\frac{35}{7}\right)}{\left(\frac{7}{7}-\frac{1}{7}\right)}=\frac{\left(-\frac{33}{7}\right)}{\left(\frac{6}{7}\right)}=-\frac{33}{6}=-\frac{11}{2}$

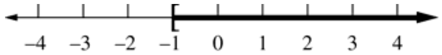
11. $\frac{\frac{11}{7}-\frac{12}{21}}{\frac{11}{7}+\frac{12}{21}}=\frac{\frac{11}{7}-\frac{4}{7}}{\frac{11}{7}+\frac{4}{7}}=\frac{\frac{7}{7}}{\frac{15}{7}}=\frac{7}{15}$

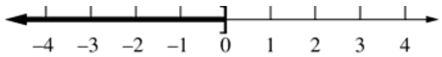
12. $\frac{\frac{1}{2}-\frac{3}{4}+\frac{7}{8}}{\frac{1}{2}+\frac{3}{4}-\frac{7}{8}}=\frac{\frac{4}{8}-\frac{6}{8}+\frac{7}{8}}{\frac{4}{8}+\frac{6}{8}-\frac{7}{8}}=\frac{\frac{5}{8}}{\frac{3}{8}}=\frac{5}{3}$

13. $1-\frac{1}{1+\frac{1}{2}}=1-\frac{1}{\frac{3}{2}}=1-\frac{2}{3}=\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$

14. $2+\frac{3}{1+\frac{5}{2}}=2+\frac{3}{\frac{2}{2}-\frac{5}{2}}=2+\frac{3}{\frac{7}{2}}$
$=2+\frac{6}{7}=\frac{14}{7}+\frac{6}{7}=\frac{20}{7}$

15. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
$=5-3=2$

e. 

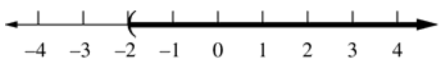
f. 

2. **a.** $(2,7)$ **b.** $[-3,4)$

c. $(-\infty,-2]$ **d.** $[-1,3]$

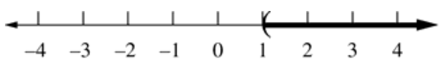
3. $x-7<2x-5$

$-2<x;(-2,\infty)$



4. $3x-5<4x-6$

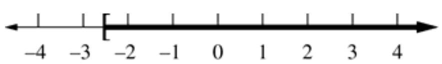
$1<x;(1,\infty)$



5. $7x-2\le 9x+3$

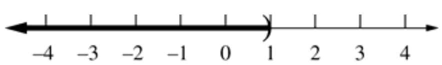
$-5\le 2x$

$x\ge -\frac{5}{2};\left[-\frac{5}{2},\infty\right)$



6. $5x-3>6x-4$

$1>x;(-\infty,1)$



7. $-4<3x+2<5$

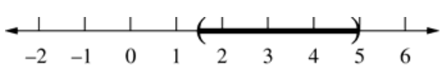
$-6<3x<3$

$-2<x<1;(-2,-1)$

8. $-3<4x-9<11$

$6<4x<20$

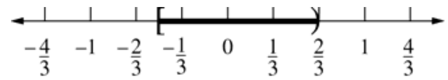
$\frac{3}{2}<x<5;\left(\frac{3}{2},5\right)$



9. $-3<1-6x\le 4$

$-4<-6x\le 3$

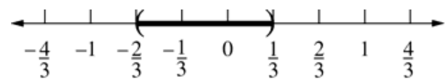
$\frac{2}{3}>x\ge -\frac{1}{2};\left[-\frac{1}{2},\frac{2}{3}\right)$



10. $4<5-3x<7$

$-1<-3x<2$

$\frac{1}{3}>x>-\frac{2}{3};\ \left(-\frac{2}{3},\frac{1}{3}\right)$



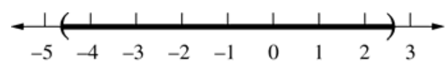
11. $x^2+2x-12<0;$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-12)}}{2(1)}=\frac{-2\pm\sqrt{52}}{2}$$

$$=-1\pm\sqrt{13}$$

$\left[x-\left(-1+\sqrt{13}\right)\right]\left[x-\left(-1-\sqrt{13}\right)\right]<0;$

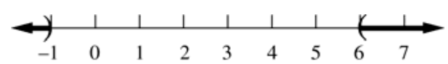
$\left(-1-\sqrt{13},-1+\sqrt{13}\right)$



12. $x^2-5x-6>0$

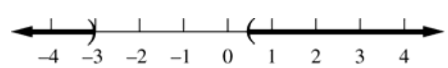
$(x+1)(x-6)>0;$

$(-\infty,-1)\cup(6,\infty)$



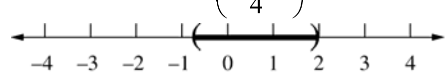
13. $2x^2+5x-3>0;\ (2x-1)(x+3)>0;$

$(-\infty,-3)\cup\left(\frac{1}{2},\infty\right)$

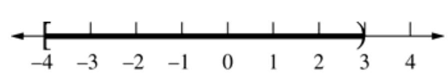


14. $4x^2-5x-6<0$

$(4x+3)(x-2)<0;\left(-\frac{3}{4},2\right)$



15. $\frac{x+4}{x-3}\le 0;\quad [-4,3)$



6. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0}\frac{[\alpha(x+h)+\beta]-(\alpha x+\beta)}{h}$
$= \lim_{h\to 0}\frac{\alpha h}{h} = \alpha$

7. $r'(x) = \lim_{h\to 0}\frac{r(x+h)-r(x)}{h}$
$= \lim_{h\to 0}\frac{[3(x+h)^2+4]-(3x^2+4)}{h}$
$= \lim_{h\to 0}\frac{6xh+3h^2}{h} = \lim_{h\to 0}(6x+3h) = 6x$

8. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0}\frac{[(x+h)^2+(x+h)+1]-(x^2+x+1)}{h}$
$= \lim_{h\to 0}\frac{2xh+h^2+h}{h} = \lim_{h\to 0}(2x+h+1) = 2x+1$

9. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0}\frac{[a(x+h)^2+b(x+h)+c]-(ax^2+bx+c)}{h}$
$= \lim_{h\to 0}\frac{2axh+ah^2+bh}{h} = \lim_{h\to 0}(2ax+ah+b)$
$= 2ax+b$

10. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0}\frac{(x+h)^4-x^4}{h}$
$= \lim_{h\to 0}\frac{4hx^3+6h^2x^2+4h^3x+h^4}{h}$
$= \lim_{h\to 0}(4x^3+6hx^2+4h^2x+h^3) = 4x^3$

11. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0}\frac{[(x+h)^3+2(x+h)^2+1]-(x^3+2x^2+1)}{h}$
$= \lim_{h\to 0}\frac{3hx^2+3h^2x+h^3+4hx+2h^2}{h}$
$= \lim_{h\to 0}(3x^2+3hx+h^2+4x+2h) = 3x^2+4x$

12. $g'(x) = \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$
$= \lim_{h\to 0}\frac{[(x+h)^4+(x+h)^2]-(x^4+x^2)}{h}$
$= \lim_{h\to 0}\frac{4hx^3+6h^2x^2+4h^3x+h^4+2hx+h^2}{h}$
$= \lim_{h\to 0}(4x^3+6hx^2+4h^2x+h^3+2x+h)$
$= 4x^3+2x$

13. $h'(x) = \lim_{h\to 0}\frac{h(x+h)-h(x)}{h}$
$= \lim_{h\to 0}\left[\left(\frac{2}{x+h}-\frac{2}{x}\right)\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{-2h}{x(x+h)}\cdot\frac{1}{h}\right] = \lim_{h\to 0}\frac{-2}{x(x+h)} = -\frac{2}{x^2}$

14. $S'(x) = \lim_{h\to 0}\frac{S(x+h)-S(x)}{h}$
$= \lim_{h\to 0}\left[\left(\frac{1}{x+h+1}-\frac{1}{x+1}\right)\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{-h}{(x+1)(x+h+1)}\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\frac{-1}{(x+1)(x+h+1)} = -\frac{1}{(x+1)^2}$

15. $F'(x) = \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$
$= \lim_{h\to 0}\left[\left(\frac{6}{(x+h)^2+1}-\frac{6}{x^2+1}\right)\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{6(x^2+1)-6(x^2+2hx+h^2+1)}{(x^2+1)(x^2+2hx+h^2+1)}\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{-12hx-6h^2}{(x^2+1)(x^2+2hx+h^2+1)}\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\frac{-12x-6h}{(x^2+1)(x^2+2hx+h^2+1)} = -\frac{12x}{(x^2+1)^2}$

16. $F'(x) = \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$
$= \lim_{h\to 0}\left[\left(\frac{x+h-1}{x+h+1}-\frac{x-1}{x+1}\right)\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{x^2+hx+h-1-(x^2+hx-h-1)}{(x+h+1)(x+1)}\cdot\frac{1}{h}\right]$
$= \lim_{h\to 0}\left[\frac{2h}{(x+h+1)(x+1)}\cdot\frac{1}{h}\right] = \frac{2}{(x+1)^2}$

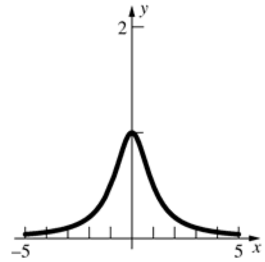
11. $h'(r) = -\frac{1}{r^2}$; $h'(r)$ is never 0; $h'(r)$ is not defined when $r = 0$, but $r = 0$ is not in the domain on $[-1, 3]$ since $h(0)$ is not defined.
Critical points: –1, 3
Note that $\lim_{x\to 0^-} h(r) = -\infty$ and $\lim_{x\to 0^+} h(x) = \infty$.
No maximum value, no minimum value.

12. $g'(x) = -\frac{2x}{(1+x^2)^2}$; $-\frac{2x}{(1+x^2)^2} = 0$ when $x = 0$
Critical points: –3, 0, 1
$g(-3) = \frac{1}{10}$, $g(0) = 1$, $g(1) = \frac{1}{2}$
Maximum value = 1, minimum value = $\frac{1}{10}$

13. $f'(x) = 4x^3 - 4x$
$= 4x(x^2 - 1)$
$= 4x(x-1)(x+1)$
$4x(x-1)(x+1) = 0$ when $x = 0, 1, -1$.
Critical points: $-2, -1, 0, 1, 2$
$f(-2) = 10$; $f(-1) = 1$; $f(0) = 2$; $f(1) = 1$;
$f(2) = 10$
Maximum value: 10
Minimum value: 1

14. $f'(x) = 5x^4 - 25x^2 + 20$
$= 5(x^4 - 5x^2 + 4)$
$= 5(x^2 - 4)(x^2 - 1)$
$= 5(x-2)(x+2)(x-1)(x+1)$
$5(x-2)(x+2)(x-1)(x+1) = 0$ when
$x = -2, -1, 1, 2$
Critical points: $-3, -2, -1, 1, 2$
$f(-3) = -79$; $f(-2) = -\frac{19}{3}$; $f(-1) = -\frac{41}{3}$;
$f(1) = \frac{35}{3}$; $f(2) = \frac{13}{3}$
Maximum value: $\frac{35}{3}$
Minimum value: -79

15. $g'(x) = -\frac{2x}{(1+x^2)^2}$; $-\frac{2x}{(1+x^2)^2} = 0$ when $x = 0$.
Critical point: 0
$g(0) = 1$
As $x \to \infty, g(x) \to 0^+$; as $x \to -\infty, g(x) \to 0^+$.
Maximum value = 1, no minimum value
(See graph.)



16. $f'(x) = \frac{1-x^2}{(1+x^2)^2}$;
$\frac{1-x^2}{(1+x^2)^2} = 0$ when $x = -1, 1$
Critical points: –1, 1, 4
$f(-1) = -\frac{1}{2}$, $f(1) = \frac{1}{2}$, $f(4) = \frac{4}{17}$
Maximum value = $\frac{1}{2}$,
minimum value = $-\frac{1}{2}$

17. $r'(\theta) = \cos\theta$; $\cos\theta = 0$ when $\theta = \frac{\pi}{2} + k\pi$
Critical points: $-\frac{\pi}{4}, \frac{\pi}{6}$
$r\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$, $r\left(\frac{\pi}{6}\right) = \frac{1}{2}$
Maximum value $= \frac{1}{2}$, minimum value $= -\frac{1}{\sqrt{2}}$

18. $s'(t) = \cos t + \sin t$; $\cos t + \sin t = 0$ when $\tan t = -1$ or $t = -\frac{\pi}{4} + k\pi$.
Critical points: $0, \frac{3\pi}{4}, \pi$
$s(0) = -1$, $s\left(\frac{3\pi}{4}\right) = \sqrt{2}$, $s(\pi) = 1$.
Maximum value $= \sqrt{2}$,
minimum value = –1

$f(x)$ is concave up on $(-\infty, 0.10) \cup (0.44, \infty)$

and concave down on $(0.10, 0.44)$;

inflection point $\approx (0.10, 0.003)$

$$\lim_{x\to\infty} \frac{5.235x^3 - 1.245x^2}{7.126x - 3.141} = \lim_{x\to\infty} \frac{5.235x^2 - 1.245x}{7.126 - \frac{3.141}{x}} = \infty$$

so $f(x)$ does not have a horizontal asymptote.

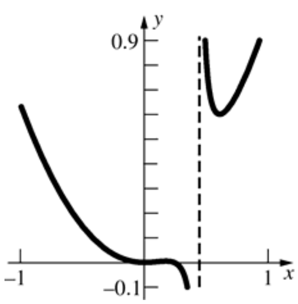
As $x \to 0.44^-, 5.235x^3 - 1.245x^2 \to 0.20$ while

$7.126x - 3.141 \to 0^-$, so $\lim_{x\to 0.44^-} f(x) = -\infty$;

as $x \to 0.44^+, 5.235x^3 - 1.245x^2 \to 0.20$ while

$7.126x - 3.141 \to 0^+$, so $\lim_{x\to 0.44^+} f(x) = \infty$;

$x \approx 0.44$ is a vertical asymptote of $f(x)$.



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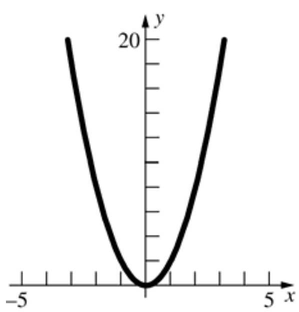

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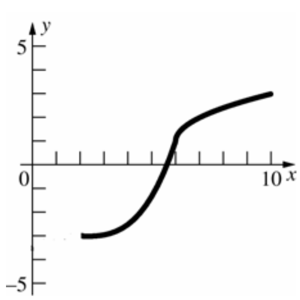

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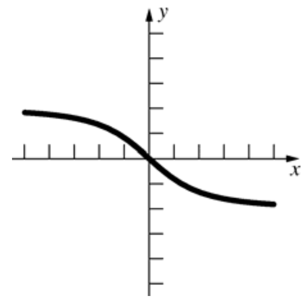

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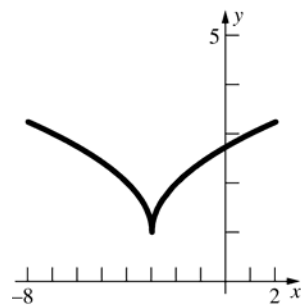

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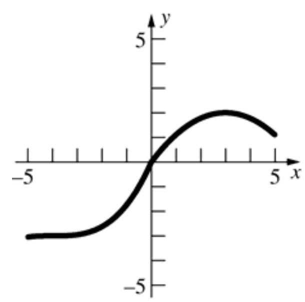

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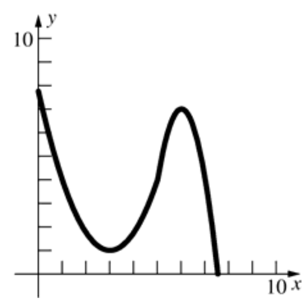

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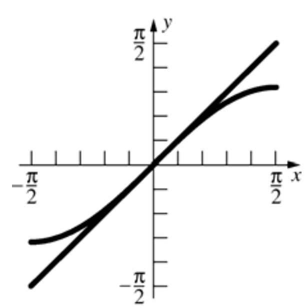

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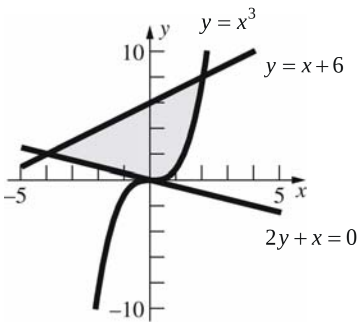


Let R_1 be the region bounded by $2y + x = 0$, $y = x + 6$, and $x = 0$.

$$A(R_1) = \int_{-4}^{0}\left[(x+6)-\left(-\frac{1}{2}x\right)\right]dx$$

$$= \int_{-4}^{0}\left(\frac{3}{2}x+6\right)dx$$

Let R_2 be the region bounded by $y = x + 6$, $y = x^3$, and $x = 0$.

$$A(R_2) = \int_0^2\left[(x+6)-x^3\right]dx = \int_0^2 (-x^3+x+6)dx$$

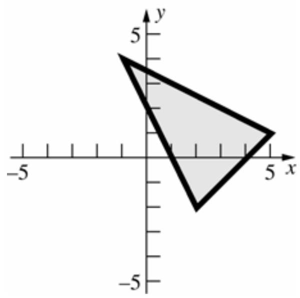
$$A(R) = A(R_1) + A(R_2)$$

$$= \int_{-4}^{0}\left(\frac{3}{2}x+6\right)dx + \int_0^2 (-x^3+x+6)dx$$

$$= \left[\frac{3}{4}x^2+6x\right]_{-4}^{0} + \left[-\frac{1}{4}x^4+\frac{1}{2}x^2+6x\right]_0^2$$

$= 12 + 10 = 22$

30.



An equation of the line through (–1, 4) and (5, 1) is $y = -\frac{1}{2}x + \frac{7}{2}$. An equation of the line through (–1, 4) and (2, –2) is $y = -2x + 2$. An equation of the line through (2, –2) and (5, 1) is $y = x - 4$. Two integrals must be used. The left-hand part of the triangle has area

$$\int_{-1}^{2}\left[-\frac{1}{2}x+\frac{7}{2}-(-2x+2)\right]dx = \int_{-1}^{2}\left(\frac{3}{2}x+\frac{3}{2}\right)dx.$$

The right-hand part of the triangle has area

$$\int_2^5\left[-\frac{1}{2}x+\frac{7}{2}-(x-4)\right]dx = \int_2^5\left(-\frac{3}{2}x+\frac{15}{2}\right)dx.$$

The triangle has area

$$\int_{-1}^{2}\left(\frac{3}{2}x+\frac{3}{2}\right)dx + \int_2^5\left(-\frac{3}{2}x+\frac{15}{2}\right)dx$$

$$= \left[\frac{3}{4}x^2+\frac{3}{2}x\right]_{-1}^{2} + \left[-\frac{3}{4}x^2+\frac{15}{2}x\right]_2^5$$

$$= \frac{27}{4}+\frac{27}{4} = \frac{27}{2} = 13.5$$

31. $\int_{-1}^{9}(3t^2-24t+36)dt = \left[t^3-12t^2+36t\right]_{-1}^{9}$ = (729 – 972 + 324) – (–1 – 12 – 36) = 130

The displacement is 130 ft. Solve $3t^2 - 24t + 36 = 0$.

$3(t-2)(t-6) = 0$

$t = 2, 6$

$$|V(t)| = \begin{cases} 3t^2-24t+36 & t \le 2, t \ge 6 \\ -3t^2+24t-36 & 2 < t < 6 \end{cases}$$

$$\int_{-1}^{9}\left|3t^2-24t+36\right|dt = \int_{-1}^{2}(3t^2-24t+36)\,dt + \int_2^6(-3t^2+24t-36)\,dt + \int_6^9(3t^2-24t+36)\,dt$$

$$= \left[t^3-12t^2+36t\right]_{-1}^{2} + \left[-t^3+12t^2-36t\right]_2^6 + \left[t^3-12t^2+36t\right]_6^9 = 81 + 32 + 81 = 194$$

The total distance traveled is 194 feet.

32. $\int_0^{3\pi/2}\left(\frac{1}{2}+\sin 2t\right)dt = \left[\frac{1}{2}t-\frac{1}{2}\cos 2t\right]_0^{3\pi/2} = \left(\frac{3\pi}{4}+\frac{1}{2}\right)-\left(0-\frac{1}{2}\right) = \frac{3\pi}{4}+1$

The displacement is $\frac{3\pi}{4}+1 \approx 3.36$ feet. Solve $\frac{1}{2}+\sin 2t = 0$ for $0 \le t \le \frac{3\pi}{2}$.

$$\sin 2t = -\frac{1}{2} \Rightarrow 2t = \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow t = \frac{7\pi}{12}, \frac{11\pi}{12}$$

32. $\int_c^{2c} \frac{x\,dx}{\sqrt{x^2+xc-2c^2}} = \int_c^{2c} \frac{x\,dx}{\sqrt{\left(x+\frac{c}{2}\right)^2-\frac{9}{4}c^2}} = \int_c^{2c} \frac{\left(x+\frac{c}{2}\right)dx}{\sqrt{\left(x+\frac{c}{2}\right)^2-\frac{9}{4}c^2}} - \frac{c}{2}\int_0^{2c} \frac{dx}{\sqrt{\left(x+\frac{c}{2}\right)^2-\frac{9}{4}c^2}}$

$= \lim_{b\to c^+}\left[\sqrt{x^2+xc-2c^2} - \frac{c}{2}\ln\left|x+\frac{c}{2}+\sqrt{x^2+xc-2c^2}\right|\right]_b^{2c}$

$= \sqrt{4c^2} - \frac{c}{2}\ln\left|\frac{5c}{2}+\sqrt{4c^2}\right| - \lim_{b\to c^+}\left[\sqrt{b^2+bc-2c^2} - \frac{c}{2}\ln\left|b+\frac{c}{2}+\sqrt{b^2+bc-2c^2}\right|\right]$

$= 2c - \frac{c}{2}\ln\frac{9c}{2} - \left(0 - \frac{c}{2}\ln\left|\frac{3c}{2}+0\right|\right) = 2c - \frac{c}{2}\ln\frac{9c}{2} + \frac{c}{2}\ln\frac{3c}{2} = 2c - \frac{c}{2}\ln 3$

33. For $0 < c < 1$, $\frac{1}{\sqrt{x}(1+x)}$ is continuous. Let $u = \frac{1}{1+x}, du = -\frac{1}{(1+x)^2}dx$.

$dv = \frac{1}{\sqrt{x}}dx, v = 2\sqrt{x}$.

$\int_c^1 \frac{1}{\sqrt{x}(1+x)}dx = \left[\frac{2\sqrt{x}}{1+x}\right]_c^1 + 2\int_c^1 \frac{\sqrt{x}dx}{(1+x)^2} = \frac{2}{2} - \frac{2\sqrt{c}}{1+c} + 2\int_c^1 \frac{\sqrt{x}dx}{(1+x)^2} = 1 - \frac{2\sqrt{c}}{1+c} + 2\int_c^1 \frac{\sqrt{x}dx}{(1+x)^2}$

Thus, $\lim_{c\to 0}\int_c^1 \frac{1}{\sqrt{x}(1+x)}dx = \lim_{c\to 0}\left[1 - \frac{2\sqrt{c}}{1+c} + 2\int_c^1 \frac{\sqrt{x}dx}{(1+x)^2}\right] = 1 - 0 + 2\int_0^1 \frac{\sqrt{x}dx}{(1+x)^2}$

This last integral is a proper integral.

34. Let $u = \frac{1}{\sqrt{1+x}}, du = -\frac{1}{2(1+x)^{3/2}}dx$

$dv = \frac{1}{\sqrt{x}}dx, v = 2\sqrt{x}$.

For $0 < c < 1$, $\int_c^1 \frac{dx}{\sqrt{x(1+x)}} = \left[\frac{2\sqrt{x}}{\sqrt{1+x}}\right]_c^1 + \int_c^1 \frac{\sqrt{x}}{(1+x)^{3/2}}dx = \frac{2\sqrt{1}}{\sqrt{2}} - \frac{2\sqrt{c}}{\sqrt{1+c}} + \int_c^1 \frac{\sqrt{x}}{(1+x)^{3/2}}dx$

Thus, $\int_0^1 \frac{dx}{\sqrt{x(1+x)}} = \lim_{c\to 0}\int_c^1 \frac{dx}{\sqrt{x(1+x)}} = \lim_{c\to 0}\left[\sqrt{2} - \frac{2\sqrt{c}}{\sqrt{1+c}} + \int_c^1 \frac{\sqrt{x}}{(1+x)^{3/2}}dx\right] = \sqrt{2} - 0 + \int_0^1 \frac{\sqrt{x}}{(1+x)^{3/2}}dx$

This is a proper integral.

35. $\int_{-3}^3 \frac{x}{\sqrt{9-x^2}}dx = \int_{-3}^0 \frac{x}{\sqrt{9-x^2}}dx + \int_0^3 \frac{x}{\sqrt{9-x^2}}dx = \lim_{b\to -3^+}\left[-\sqrt{9-x^2}\right]_b^0 + \lim_{b\to 3^-}\left[-\sqrt{9-x^2}\right]_0^b$

$= -\sqrt{9} + \lim_{b\to -3^+}\sqrt{9-b^2} - \lim_{b\to 3^-}\sqrt{9-b^2} + \sqrt{9} = -3 + 0 - 0 + 3 = 0$

36. $\int_{-3}^3 \frac{x}{9-x^2}dx = \int_{-3}^0 \frac{x}{9-x^2}dx + \int_0^3 \frac{x}{9-x^2}dx = \lim_{b\to 3^+}\left[-\frac{1}{2}\ln\left|9-x^2\right|\right]_b^0 + \lim_{b\to 3^-}\left[-\frac{1}{2}\ln\left|9-x^2\right|\right]_0^b$

$= -\ln 3 + \lim_{b\to -3^+}\frac{1}{2}\ln\left|9-b^2\right| - \lim_{b\to 3^-}\frac{1}{2}\ln\left|9-b^2\right| + \ln 3 = (-\ln 3 - \infty) + (\infty + \ln 3)$

The integral diverges.

12. $LQ'' + RQ' + \frac{Q}{C} = E;\ 10^{-2}Q'' + \frac{Q}{10^{-7}} = 20;\ Q'' + 10^9 Q = 2000$

The auxiliary equation, $r^2 + 10^9 = 0$, has roots $\pm 10^{9/2} i$.

$Q_h = C_1 \cos 10^{9/2} t + C_2 \sin 10^{9/2} t$

$Q_p = 2000(10^{-9}) = 2(10^{-6})$ is a particular solution (by inspection).

General solution: $Q(t) = 2(10^{-6}) + C_1 \cos 10^{9/2} t + C_2 \sin 10^{9/2} t$

Then $I(t) = Q'(t) = -10^{9/2} C_1 \sin 10^{9/2} t + 10^{9/2} C_2 \cos 10^{9/2} t$.

If $t = 0$, $Q = 0$, $I = 0$, then $0 = 2(10^{-6}) + C_1$ and $0 = C_2$.

Therefore, $I(t) = -10^{9/2}(-2[10^{-6}]) \sin 10^{9/2} t = 2(10^{-3/2}) \sin 10^{9/2} t$.

13. $3.5Q'' + 1000Q + \frac{Q}{[2(10^{-6})]} = 120 \sin 377t$

(Values are approximated to 6 significant figures for the remainder of the problem.)

$Q'' + 285.714Q' + 142857Q = 34.2857 \sin 377t$

Roots of the auxiliary equation are
$-142.857 \pm 349.927i$.

$Q_h = e^{-142.857t}(C_1 \cos 349.927t + C_2 \sin 349.927t)$

$Q_p = -3.18288(10^{-4}) \cos 377t + 2.15119(10^{-6}) \sin 377t$

Then, $Q = -3.18288(10^{-4}) \cos 377t + 2.15119(10^{-6}) \sin 377t + Q_h$.

$I = Q' = 0.119995 \sin 377t + 0.000810998 \cos 377t + Q_h'$

0.000888 cos $377t$ is small and $Q_h' \to 0$ as $t \to \infty$, so the steady-state current is $I \approx 0.12 \sin 377t$.

14. a. Roots of the auxiliary equation are $\pm Bi$.

$y_h = C_1 \cos Bt + C_2 \sin Bt$.

$y_p = \left[\frac{c}{(B^2 - A^2)}\right] \sin At$

The desired result follows.

b. $y_p = \left(-\frac{c}{2B}\right) t \cos Bt$, so

$y = C_1 \cos Bt + C_2 \sin Bt - \left(\frac{c}{2B}\right) t \cos Bt$.

c. Due to the t factor in the last term, it increases without bound.

15. $A \sin(\beta t + \gamma) = A(\sin \beta t \cos \gamma + \cos \beta t \sin \gamma)$

$= (A \cos \gamma) \sin \beta t + (A \sin \gamma) \cos \beta t$

$= C_1 \sin \beta t + C_2 \cos \beta t$, where $C_1 = A \cos \gamma$ and $C_2 = A \sin \gamma$.

[Note that
$C_1^2 + C_2^2 = A^2 \cos^2 \gamma + A^2 \sin^2 \gamma = A^2$.)

16. The first two terms have period $\frac{2\pi}{B}$ and the last has period $\frac{2\pi}{A}$. Then the sum of the three terms is periodic if $m\left(\frac{2\pi}{B}\right) = n\left(\frac{2\pi}{B}\right)$ for some integers m, n; equivalently, if $\frac{B}{A} = \frac{m}{n}$, a rational number.