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1 PRECALCULUS REVIEW

1.1 Real Numbers, Functions, and Graphs

Preliminary Questions

1. Give an example of numbers a and b such that $a < b$ and $|a| > |b|$.

SOLUTION Take $a = -3$ and $b = 1$. Then $a < b$ but $|a| = 3 > 1 = |b|$.

2. Which numbers satisfy $|a| = a$? Which satisfy $|a| = -a$? What about $|-a| = a$?

SOLUTION The numbers $a \geq 0$ satisfy $|a| = a$ and $|-a| = a$. The numbers $a \leq 0$ satisfy $|a| = -a$.

3. Give an example of numbers a and b such that
$|a + b| < |a| + |b|$.

SOLUTION Take $a = -3$ and $b = 1$. Then

$$|a + b| = |-3 + 1| = |-2| = 2, \qquad \text{but} \qquad |a| + |b| = |-3| + |1| = 3 + 1 = 4.$$

Thus, $|a + b| < |a| + |b|$.

4. Are there numbers a and b such that $|a + b| > |a| + |b|$?

SOLUTION No. By the Triangle inequality, $|a + b| \leq |a| + |b|$ for all real numbers a and b.

5. What are the coordinates of the point lying at the intersection of the lines $x = 9$ and $y = -4$?

SOLUTION The point $(9, -4)$ lies at the intersection of the lines $x = 9$ and $y = -4$.

6. In which quadrant do the following points lie?

(a) $(1, 4)$ **(b)** $(-3, 2)$ **(c)** $(4, -3)$ **(d)** $(-4, -1)$

SOLUTION

(a) Because both the x- and y-coordinates of the point $(1, 4)$ are positive, the point $(1, 4)$ lies in the first quadrant.

(b) Because the x-coordinate of the point $(-3, 2)$ is negative but the y-coordinate is positive, the point $(-3, 2)$ lies in the second quadrant.

(c) Because the x-coordinate of the point $(4, -3)$ is positive but the y-coordinate is negative, the point $(4, -3)$ lies in the fourth quadrant.

(d) Because both the x- and y-coordinates of the point $(-4, -1)$ are negative, the point $(-4, -1)$ lies in the third quadrant.

7. What is the radius of the circle with equation
$(x - 7)^2 + (y - 8)^2 = 9$?

SOLUTION The circle with equation $(x - 7)^2 + (y - 8)^2 = 9$ has radius 3.

8. The equation $f(x) = 5$ has a solution if (choose one):

(a) 5 belongs to the domain of f.

(b) 5 belongs to the range of f.

SOLUTION The correct response is **(b)**: the equation $f(x) = 5$ has a solution if 5 belongs to the range of f.

9. What kind of symmetry does the graph have if $f(-x) = -f(x)$?

SOLUTION If $f(-x) = -f(x)$, then the graph of f is symmetric with respect to the origin.

10. Is there a function that is both even and odd?

SOLUTION Yes. The constant function $f(x) = 0$ for all real numbers x is both even and odd because

$$f(-x) = 0 = f(x)$$

and

$$f(-x) = 0 = -0 = -f(x)$$

for all real numbers x.

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71. Suppose that f has domain [4, 8] and range [2, 6]. Find the domain and range of:

(a) $y = f(x) + 3$ **(b)** $y = f(x + 3)$

(c) $y = f(3x)$ **(d)** $y = 3f(x)$

SOLUTION

(a) $f(x) + 3$ is obtained by shifting $f(x)$ upward three units. Therefore, the domain remains [4, 8], while the range becomes [5, 9].

(b) $f(x + 3)$ is obtained by shifting $f(x)$ left three units. Therefore, the domain becomes [1, 5], while the range remains [2, 6].

(c) $f(3x)$ is obtained by compressing $f(x)$ horizontally by a factor of three. Therefore, the domain becomes $[\frac{4}{3}, \frac{8}{3}]$, while the range remains [2, 6].

(d) $3f(x)$ is obtained by stretching $f(x)$ vertically by a factor of three. Therefore, the domain remains [4, 8], while the range becomes [6, 18].

73. Suppose that the graph of $f(x) = \sin x$ is compressed horizontally by a factor of 2 and then shifted 5 units to the right.

(a) What is the equation for the new graph?

(b) What is the equation if you first shift by 5 and then compress by 2?

(c) $\boxed{\text{GU}}$ Verify your answers by plotting your equations.

SOLUTION

(a) Let $f(x) = \sin x$. After compressing the graph of f horizontally by a factor of 2, we obtain the function $g(x) = f(2x) = \sin 2x$. Shifting the graph 5 units to the right then yields

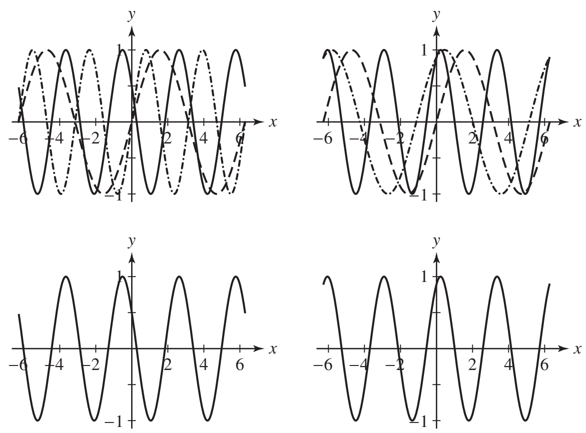
$$h(x) = g(x - 5) = \sin 2(x - 5) = \sin(2x - 10).$$

(b) Let $f(x) = \sin x$. After shifting the graph 5 units to the right, we obtain the function $g(x) = f(x - 5) = \sin(x - 5)$. Compressing the graph horizontally by a factor of 2 then yields

$$h(x) = g(2x) = \sin(2x - 5).$$

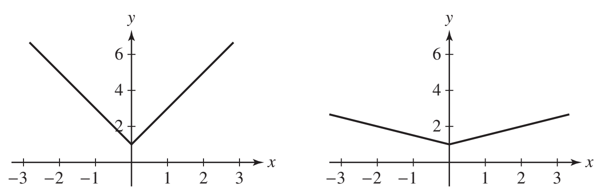
(c) The figure below at the top left shows the graphs of $y = \sin x$ (the dashed curve), the sine graph compressed horizontally by a factor of 2 (the dash, double dot curve) and then shifted right 5 units (the solid curve). Compare this last graph with the graph of $y = \sin(2x - 10)$ shown at the bottom left.

The figure below at the top right shows the graphs of $y = \sin x$ (the dashed curve), the sine graph shifted to the right 5 units (the dash, double dot curve) and then compressed horizontally by a factor of 2 (the solid curve). Compare this last graph with the graph of $y = \sin(2x - 5)$ shown at the bottom right.



75. Sketch the graph of $y = f(2x)$ and $y = f\left(\frac{1}{2}x\right)$, where $f(x) = |x| + 1$ (Figure 28).

SOLUTION The graph of $y = f(2x)$ is obtained by compressing the graph of $y = f(x)$ horizontally by a factor of 2 (see the graph below on the left). The graph of $y = f(\frac{1}{2}x)$ is obtained by stretching the graph of $y = f(x)$ horizontally by a factor of 2 (see the graph below on the right).



$f(2x)$ $f(x/2)$

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7. $\displaystyle\lim_{x\to 2}\frac{x^x-4}{x^2-4}$

SOLUTION Let $f(x)=\frac{x^x-4}{x^2-4}$. The data in the table below suggests that

$$\lim_{x\to 2}\frac{x^x-4}{x^2-4}\approx 1.69.$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	1.575461	1.680633	1.691888	1.694408	1.705836	1.828386

(The exact value is $1+\ln 2$.)

9. $\displaystyle\lim_{x\to 1}\left(\frac{7}{1-x^7}-\frac{3}{1-x^3}\right)$

SOLUTION Let $f(x)=\frac{7}{1-x^7}-\frac{3}{1-x^3}$. The data in the table below suggests that

$$\lim_{x\to 1}\left(\frac{7}{1-x^7}-\frac{3}{1-x^3}\right)\approx 2.00.$$

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.347483	2.033498	2.003335	1.996668	1.966835	1.685059

(The exact value is 2.)

In Exercises 11–50, evaluate the limit if it exists. If not, determine whether the one-sided limits exist (finite or infinite).

11. $\displaystyle\lim_{x\to 4}(3+x^{1/2})$

SOLUTION $\displaystyle\lim_{x\to 4}(3+x^{1/2})=3+\sqrt{4}=5.$

13. $\displaystyle\lim_{x\to -2}\frac{4}{x^3}$

SOLUTION $\displaystyle\lim_{x\to -2}\frac{4}{x^3}=\frac{4}{(-2)^3}=-\frac{1}{2}.$

15. $\displaystyle\lim_{t\to 9}\frac{\sqrt{t}-3}{t-9}$

SOLUTION $\displaystyle\lim_{t\to 9}\frac{\sqrt{t}-3}{t-9}=\lim_{t\to 9}\frac{\sqrt{t}-3}{(\sqrt{t}-3)(\sqrt{t}+3)}=\lim_{t\to 9}\frac{1}{\sqrt{t}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}.$

17. $\displaystyle\lim_{x\to 1}\frac{x^3-x}{x-1}$

SOLUTION $\displaystyle\lim_{x\to 1}\frac{x^3-x}{x-1}=\lim_{x\to 1}\frac{x(x-1)(x+1)}{x-1}=\lim_{x\to 1}x(x+1)=1(1+1)=2.$

19. $\displaystyle\lim_{t\to 9}\frac{t-6}{\sqrt{t}-3}$

SOLUTION Because the one-sided limits

$$\lim_{t\to 9-}\frac{t-6}{\sqrt{t}-3}=-\infty \qquad \text{and} \qquad \lim_{t\to 9+}\frac{t-6}{\sqrt{t}-3}=\infty,$$

are not equal, the two-sided limit

$$\lim_{t\to 9}\frac{t-6}{\sqrt{t}-3} \qquad \text{does not exist.}$$

21. $\displaystyle\lim_{x\to -1^+}\frac{1}{x+1}$

SOLUTION As $x\to -1^+$, $x+1\to 0^+$. Therefore,

$$\lim_{x\to -1^+}\frac{1}{x+1}=\infty.$$

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In Exercises 35–37, use Theorem 1 to verify the formula.

35. $\frac{d}{dx}\cot x = -\csc^2 x$

SOLUTION $\cot x = \frac{\cos x}{\sin x}$. Using the quotient rule and the derivative formulas, we compute:

$$\frac{d}{dx}\cot x = \frac{d}{dx}\frac{\cos x}{\sin x} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x.$$

37. $\frac{d}{dx}\csc x = -\csc x \cot x$

SOLUTION Since $\csc x = \frac{1}{\sin x}$, we can apply the quotient rule and the two known derivatives to get:

$$\frac{d}{dx}\csc x = \frac{d}{dx}\frac{1}{\sin x} = \frac{\sin x(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\frac{1}{\sin x} = -\cot x \csc x.$$

In Exercises 39–42, calculate the higher derivative.

39. $f''(\theta)$, $\quad f(\theta) = \theta \sin\theta$

SOLUTION Let $f(\theta) = \theta\sin\theta$. Then

$$f'(\theta) = \theta\cos\theta + \sin\theta$$
$$f''(\theta) = \theta(-\sin\theta) + \cos\theta + \cos\theta = -\theta\sin\theta + 2\cos\theta.$$

41. y'', $\quad y'''$, $\quad y = \tan x$

SOLUTION Let $y = \tan x$. Then $y' = \sec^2 x$ and by the Chain Rule,

$$y'' == \frac{d}{dx}\sec^2 x = 2(\sec x)(\sec x \tan x) = 2\sec^2 x \tan x$$
$$y''' = 2\sec^2 x(\sec^2 x) + (4\sec^2 x \tan x)\tan x = 2\sec^4 x + 4\sec^2 x \tan^2 x$$

43. Calculate the first five derivatives of $f(x) = \cos x$. Then determine $f^{(8)}(x)$ and $f^{(37)}(x)$.

SOLUTION Let $f(x) = \cos x$.

- Then $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, $f^{(4)}(x) = \cos x$, and $f^{(5)}(x) = -\sin x$.
- Accordingly, the successive derivatives of f cycle among

$$\{-\sin x, -\cos x, \sin x, \cos x\}$$

  in that order. Since 8 is a multiple of 4, we have $f^{(8)}(x) = \cos x$.
- Since 36 is a multiple of 4, we have $f^{(36)}(x) = \cos x$. Therefore, $f^{(37)}(x) = -\sin x$.

45. Find the values of x between 0 and 2π where the tangent line to the graph of $y = \sin x \cos x$ is horizontal.

SOLUTION Let $y = \sin x \cos x$. Then

$$y' = (\sin x)(-\sin x) + (\cos x)(\cos x) = \cos^2 x - \sin^2 x.$$

When $y' = 0$, we have $\sin x = \pm\cos x$. In the interval $[0, 2\pi]$, this occurs when $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

47. GU Let $g(t) = t - \sin t$.

(a) Plot the graph of g with a graphing utility for $0 \le t \le 4\pi$.

(b) Show that the slope of the tangent line is nonnegative. Verify this on your graph.

(c) For which values of t in the given range is the tangent line horizontal?

SOLUTION Let $g(t) = t - \sin t$.

(a) Here is a graph of g over the interval $[0, 4\pi]$.

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41. $y = \dfrac{x^4 + \sqrt{x}}{x^2}$

SOLUTION Let

$$y = \frac{x^4 + \sqrt{x}}{x^2} = x^2 + x^{-3/2}.$$

Then

$$\frac{dy}{dx} = 2x - \frac{3}{2}x^{-5/2}.$$

43. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

SOLUTION Let $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$. Then

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2} \frac{d}{dx}\left(x + \sqrt{x + \sqrt{x}}\right)\\ &= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\frac{d}{dx}\left(x + \sqrt{x}\right)\right)\\ &= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right).\end{aligned}$$

45. $y = \tan(t^{-3})$

SOLUTION Let $y = \tan(t^{-3})$. Then

$$\frac{dy}{dt} = \sec^2(t^{-3})\frac{d}{dt}t^{-3} = -3t^{-4}\sec^2(t^{-3}).$$

47. $y = \sin(2x)\cos^2 x$

SOLUTION Let $y = \sin(2x)\cos^2 x = 2\sin x\cos^3 x$. Then

$$\frac{dy}{dx} = -6\sin^2 x\cos^2 x + 2\cos^4 x.$$

49. $y = \dfrac{t}{1 + \sec t}$

SOLUTION Let $y = \dfrac{t}{1 + \sec t}$. Then

$$\frac{dy}{dt} = \frac{1 + \sec t - t\sec t\tan t}{(1 + \sec t)^2}.$$

51. $y = \dfrac{8}{1 + \cot\theta}$

SOLUTION Let $y = \dfrac{8}{1 + \cot\theta} = 8(1 + \cot\theta)^{-1}$. Then

$$\frac{dy}{d\theta} = -8(1 + \cot\theta)^{-2}\frac{d}{d\theta}(1 + \cot\theta) = \frac{8\csc^2\theta}{(1 + \cot\theta)^2}.$$

53. $y = \tan(\sqrt{1 + \csc\theta})$

SOLUTION

$$\begin{aligned}\frac{dy}{dx} &= \sec^2(\sqrt{1 + \csc\theta})\frac{d}{dx}\sqrt{1 + \csc\theta} = \sec^2(\sqrt{1 + \csc\theta}) \cdot \frac{1}{2}(1 + \csc\theta)^{-1/2}\frac{d}{dx}(1 + \csc\theta)\\ &= -\frac{\sec^2(\sqrt{1 + \csc\theta})\csc\theta\cot\theta}{2(\sqrt{1 + \csc\theta})}.\end{aligned}$$

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59. A rectangular box of height h with a square base of side b has volume $V = 4$ m^3. Two of the side faces are made of material costing \$40/m^2. The remaining sides cost \$20/m^2. Which values of b and h minimize the cost of the box?

SOLUTION Because the volume of the box is

$$V = b^2h = 4 \quad \text{it follows that} \quad h = \frac{4}{b^2}.$$

Now, the cost of the box is

$$C = 40(2bh) + 20(2bh) + 20b^2 = 120bh + 20b^2 = \frac{480}{b} + 20b^2.$$

Thus,

$$C'(b) = -\frac{480}{b^2} + 40b = 0$$

when $b = \sqrt[3]{12}$ meters. Because $C(b) \to \infty$ as $b \to 0+$ and as $b \to \infty$, it follows that cost is minimized when $b = \sqrt[3]{12}$ meters and $h = \frac{1}{3}\sqrt[3]{12}$ meters.

61. Let $N(t)$ be the size of a tumor (in units of 10^6 cells) at time t (in days). According to the **Gompertz Model**, $dN/dt = N(a - b\ln N)$ where a, b are positive constants. Show that the maximum value of N is $e^{\frac{a}{b}}$ and that the tumor increases most rapidly when $N = e^{\frac{a}{b}-1}$.

SOLUTION Given $dN/dt = N(a - b\ln N)$, the critical points of N occur when $N = 0$ and when $N = e^{a/b}$. The sign of $N'(t)$ changes from positive to negative at $N = e^{a/b}$ so the maximum value of N is $e^{a/b}$. To determine when N changes most rapidly, we calculate

$$N''(t) = N\left(-\frac{b}{N}\right) + a - b\ln N = (a - b) - b\ln N.$$

Thus, $N'(t)$ is increasing for $N < e^{a/b-1}$, is decreasing for $N > e^{a/b-1}$ and is therefore maximum when $N = e^{a/b-1}$. Therefore, the tumor increases most rapidly when $N = e^{\frac{a}{b}-1}$.

63. Find the maximum volume of a right-circular cone placed upside-down in a right-circular cone of radius $R = 3$ and height $H = 4$ as in Figure 3. A cone of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.

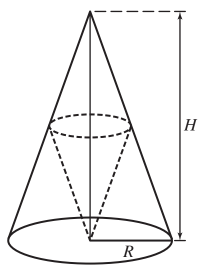

FIGURE 3

SOLUTION Let r denote the radius and h the height of the upside down cone. By similar triangles, we obtain the relation

$$\frac{4-h}{r} = \frac{4}{3} \qquad \text{so} \qquad h = 4\left(1 - \frac{r}{3}\right)$$

and the volume of the upside down cone is

$$V(r) = \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi\left(r^2 - \frac{r^3}{3}\right)$$

for $0 \le r \le 3$. Thus,

$$\frac{dV}{dr} = \frac{4}{3}\pi\left(2r - r^2\right),$$

and the critical points are $r = 0$ and $r = 2$. Because $V(0) = V(3) = 0$ and

$$V(2) = \frac{4}{3}\pi\left(4 - \frac{8}{3}\right) = \frac{16}{9}\pi,$$

the maximum volume of a right-circular cone placed upside down in a right-circular cone of radius 3 and height 4 is

$$\frac{16}{9}\pi.$$

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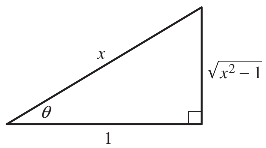
51. Compute $\int \frac{dx}{x^2-1}$ in two ways and verify that the answers agree: first via trigonometric substitution and then using the identity

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

SOLUTION Using trigonometric substitution, let $x = \sec\theta$. Then $dx = \sec\theta\tan\theta d\theta$, $x^2 - 1 = \sec^2\theta - 1 = \tan^2\theta$, and

$$I = \int \frac{dx}{x^2-1} = \int \frac{\sec\theta\tan\theta\,d\theta}{\tan^2\theta} = \int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \frac{d\theta}{\sin\theta} = \int \csc\theta\,d\theta = \ln|\csc\theta - \cot\theta| + C.$$

Since $x = \sec\theta$, we construct the following right triangle:



From this we see that $\csc\theta = x/\sqrt{x^2-1}$ and $\cot\theta = 1/\sqrt{x^2-1}$. This gives us

$$I = \ln\left|\frac{x}{\sqrt{x^2-1}} - \frac{1}{\sqrt{x^2-1}}\right| + C = \ln\left|\frac{x-1}{\sqrt{x^2-1}}\right| + C.$$

Using the given identity, we get

$$I = \int \frac{dx}{x^2-1} = \frac{1}{2}\int\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx = \frac{1}{2}\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{dx}{x+1} = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C.$$

To confirm that these answers agree, note that

$$\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| = \ln\sqrt{\left|\frac{x-1}{x+1}\right|} = \ln\left|\frac{\sqrt{x-1}}{\sqrt{x+1}}\cdot\frac{\sqrt{x-1}}{\sqrt{x-1}}\right| = \ln\left|\frac{x-1}{\sqrt{x^2-1}}\right|.$$

53. A charged wire creates an electric field at a point P located at a distance D from the wire (Figure 8). The component $E_\perp$ of the field perpendicular to the wire (in newtons per coulomb) is

$$E_\perp = \int_{x_1}^{x_2} \frac{k\lambda D}{(x^2+D^2)^{3/2}}\,dx$$

where λ is the charge density (coulombs per meter), $k = 8.99 \times 10^9$ N·m²/C² (Coulomb constant), and x_1, x_2 are as in the figure. Suppose that $\lambda = 6 \times 10^{-4}$ C/m, and $D = 3$ m. Find $E_\perp$ if (a) $x_1 = 0$ and $x_2 = 30$ m, and (b) $x_1 = -15$ m and $x_2 = 15$ m.

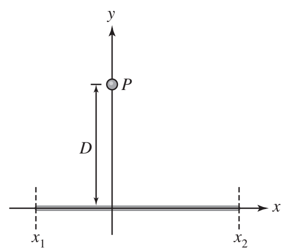


FIGURE 8

SOLUTION Let $x = D\tan\theta$. Then $dx = D\sec^2\theta\,d\theta$,

$$x^2 + D^2 = D^2\tan^2\theta + D^2 = D^2(\tan^2\theta + 1) = D^2\sec^2\theta,$$

and

$$E_\perp = \int_{x_1}^{x_2} \frac{k\lambda D}{(x^2+D^2)^{3/2}}\,dx = k\lambda D\int_{x_1}^{x_2}\frac{D\sec^2\theta\,d\theta}{(D^2\sec^2\theta)^{3/2}}$$
$$= \frac{k\lambda D^2}{D^3}\int_{x_1}^{x_2}\frac{\sec^2\theta\,d\theta}{\sec^3\theta} = \frac{k\lambda}{D}\int_{x_1}^{x_2}\cos\theta\,d\theta = \frac{k\lambda}{D}\sin\theta\Big|_{x_1}^{x_2}$$

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4. Which is the projection of $\mathbf{v}$ along $\mathbf{v}$: (a) $\mathbf{v}$ or (b) $\mathbf{e_v}$?

SOLUTION The projection of $\mathbf{v}$ along itself is $\mathbf{v}$, since

$$\mathbf{v}_{||\mathbf{v}} = \left(\frac{\mathbf{v}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = \mathbf{v}.$$

5. Let $\mathbf{u}_{||\mathbf{v}}$ be the projection of $\mathbf{u}$ along $\mathbf{v}$. Which of the following is the projection $\mathbf{u}$ along the vector $2\mathbf{v}$ and which is the projection of $2\mathbf{u}$ along $\mathbf{v}$?

(a) $\frac{1}{2}\mathbf{u}_{||\mathbf{v}}$ **(b)** $\mathbf{u}_{||\mathbf{v}}$ **(c)** $2\mathbf{u}_{||\mathbf{v}}$

SOLUTION Since $\mathbf{u}_{\|\mathbf{v}}$ is the projection of $\mathbf{u}$ along $\mathbf{v}$, we have,

$$\mathbf{u}_{\|\mathbf{v}} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$$

The projection of $\mathbf{u}$ along the vector $2\mathbf{v}$ is

$$\mathbf{u}_{\|2\mathbf{v}} = \left(\frac{\mathbf{u}\cdot(2\mathbf{v})}{(2\mathbf{v})\cdot(2\mathbf{v})}\right)2\mathbf{v} = \left(\frac{2(\mathbf{u}\cdot\mathbf{v})}{4(\mathbf{v}\cdot\mathbf{v})}\right)2\mathbf{v} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = \mathbf{u}_{\|\mathbf{v}}.$$

That is, $\mathbf{u}_{\|\mathbf{v}}$ is the projection of $\mathbf{u}$ along $2\mathbf{v}$. Notice that the projection of $\mathbf{u}$ along $\mathbf{v}$ is the projection of $\mathbf{u}$ along the unit vector $\mathbf{e_v}$, hence it depends on the direction of $\mathbf{v}$ rather than on the length of $\mathbf{v}$. Therefore, the projection of $\mathbf{u}$ along $\mathbf{v}$ and along $2\mathbf{v}$ is the same vector.

For the second question,

$$(2\mathbf{u})_{\|\mathbf{v}} = \left(\frac{(2\mathbf{u})\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = 2\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = 2\mathbf{u}_{\|\mathbf{v}}.$$

That is, the projection of $2\mathbf{u}$ along $\mathbf{v}$ is twice the projection of $\mathbf{u}$ along $\mathbf{v}$.

6. Which of the following is equal to $\cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$?

(a) $\mathbf{u}\cdot\mathbf{v}$ **(b)** $\mathbf{u}\cdot\mathbf{e_v}$ **(c)** $\mathbf{e_u}\cdot\mathbf{e_v}$

SOLUTION By the Theorems on the Dot Product and the Angle Between Vectors, we have

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\mathbf{u}}{\|\mathbf{u}\|}\cdot\frac{\mathbf{v}}{\|\mathbf{v}\|} = \mathbf{e_u}\cdot\mathbf{e_v}$$

The correct answer is (c).

Exercises

In Exercises 1–12, compute the dot product.

1. $\langle 1, 2, 1\rangle \cdot \langle 4, 3, 5\rangle$

SOLUTION Using the definition of the dot product we obtain

$$\langle 1, 2, 1\rangle \cdot \langle 4, 3, 5\rangle = 1\cdot 4 + 2\cdot 3 + 1\cdot 5 = 15$$

3. $\langle 0, 1, 0\rangle \cdot \langle 7, 41, -3\rangle$

SOLUTION The dot product is

$$\langle 0, 1, 0\rangle \cdot \langle 7, 41, -3\rangle = 0\cdot 7 + 1\cdot 41 + 0\cdot(-3) = 41$$

5. $\langle 3, 1\rangle \cdot \langle 4, -7\rangle$

SOLUTION The dot product of the two vectors is the following scalar:

$$\langle 3, 1\rangle \cdot \langle 4, -7\rangle = 3\cdot 4 + 1\cdot(-7) = 5$$

7. $\mathbf{k}\cdot\mathbf{j}$

SOLUTION By the orthogonality of $\mathbf{j}$ and $\mathbf{k}$, we have $\mathbf{k}\cdot\mathbf{j} = 0$

9. $(\mathbf{i}+\mathbf{j})\cdot(\mathbf{j}+\mathbf{k})$

SOLUTION By the distributive law and the orthogonality of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ we have

$$(\mathbf{i}+\mathbf{j})\cdot(\mathbf{j}+\mathbf{k}) = \mathbf{i}\cdot\mathbf{j} + \mathbf{i}\cdot\mathbf{k} + \mathbf{j}\cdot\mathbf{j} + \mathbf{j}\cdot\mathbf{k} = 0 + 0 + 1 + 0 = 1$$

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In Exercises 41–44, compute the given partial derivatives.

41. $f(x, y) = 3x^2y + 4x^3y^2 - 7xy^5, \quad f_x(1, 2)$

SOLUTION Differentiating with respect to x gives

$$f_x(x, y) = 6xy + 12x^2y^2 - 7y^5$$

Evaluating at (1, 2) gives

$$f_x(1, 2) = 6 \cdot 1 \cdot 2 + 12{\cdot}1^2 \cdot 2^2 - 7 \cdot 2^5 = -164.$$

43. $g(u, v) = u \ln(u + v), \quad g_u(1, 2)$

SOLUTION Using the Product Rule and the Chain Rule we get

$$g_u(u, v) = \frac{\partial}{\partial u}(u \ln(u + v)) = 1 \cdot \ln(u + v) + u \cdot \frac{1}{u + v} = \ln(u + v) + \frac{u}{u + v}$$

At the point (1, 2) we have

$$g_u(1, 2) = \ln(1 + 2) + \frac{1}{1 + 2} = \ln 3 + \frac{1}{3}.$$

Exercises 45 and 46 refer to Example 5.

45. Calculate N for $L = 0.4$, $R = 0.12$, and $d = 10$, and use the linear approximation to estimate ΔN if d is increased from 10 to 10.4.

SOLUTION From the example in the text we have

$$N = \left(\frac{2200R}{Ld}\right)^{1.9}$$

Calculating N for $L = 0.4$, $R = 0.12$, and $d = 10$ we have

$$N = \left(\frac{2200 \cdot 0.12}{0.4 \cdot 10}\right)^{1.9} \approx 2865.058$$

then we will use the derivation

$$\Delta N \approx \frac{\partial N}{\partial d}\Delta d$$

since d is increasing from 10 to 10.4. We need to compute $\partial N/\partial d$, with L and R constant:

$$\begin{aligned}\frac{\partial N}{\partial d} &= \frac{\partial}{\partial d}\left(\frac{2200R}{Ld}\right)^{1.9} \\ &= \left(\frac{2200R}{L}\right)^{1.9} \frac{\partial}{\partial d}(d^{-1.9}) \\ &= -1.9\left(\frac{2200R}{L}\right)^{1.9} d^{-2.9}\end{aligned}$$

we have first

$$\left.\frac{\partial N}{\partial d}\right|_{(L,R,d)=(0.4,0.12,10)} = -1.9\left(\frac{2200 \cdot 0.12}{0.4}\right)^{1.9} (10)^{-2.9} \approx -544.361$$

Therefore we can conclude:

$$\Delta N \approx \frac{\partial N}{\partial d}\Delta d \approx (-544.361)(10.4 - 10) = -217.744$$

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The rotated graph is $z = g(y) = \sqrt{a^2 - (y-b)^2}$, $b - a \le y \le b + a$. So, we have,

$$g'(y) = \frac{-2(y-b)}{2\sqrt{a^2-(y-b)^2}} = -\frac{y-b}{\sqrt{a^2-(y-b)^2}}$$

$$\sqrt{1+g'(y)^2} = \sqrt{1+\frac{(y-b)^2}{a^2-(y-b)^2}} = \sqrt{\frac{a^2-(y-b)^2+(y-b)^2}{a^2-(y-b)^2}} = \frac{a}{\sqrt{a^2-(y-b)^2}}$$

We now use symmetry and Eq. (14) to obtain the following area of the torus (we assume that $b - a > 0$, hence $y > 0$):

$$\text{Area}(\mathbf{T}) = 2\cdot 2\pi \int_{b-a}^{b+a} |y|\sqrt{1+g'(y)^2}\,dy = 4\pi\int_{b-a}^{b+a}\frac{ay}{\sqrt{a^2-(y-b)^2}}\,dy \tag{1}$$

(b) We compute the integral using the substitution $u = \frac{y-b}{a}$, $du = \frac{1}{a}\,dy$. We get:

$$\int_{b-a}^{b+a}\frac{ay}{\sqrt{a^2-(y-b)^2}}\,dy = \int_{-1}^{1}\frac{a^2u+ab}{\sqrt{a^2-a^2u^2}}a\,du = \int_{-1}^{1}\frac{a^2u+ab}{\sqrt{1-u^2}}\,du$$

$$= \int_{-1}^{1}\frac{a^2u}{\sqrt{1-u^2}}\,du + \int_{-1}^{1}\frac{ab}{\sqrt{1-u^2}}\,du$$

The first integral is zero since the integrand is an odd function. We get:

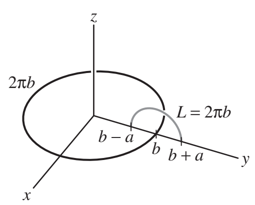
$$\int_{b-a}^{b+a}\frac{ay}{\sqrt{a^2-(y-b)^2}}\,dy = 2\int_0^1\frac{ab}{\sqrt{1-u^2}}\,du = 2ab\sin^{-1}u\Big|_0^1 = 2ab\left(\frac{\pi}{2}-0\right) = \pi ab$$

Substituting in (1) gives the following area:

$$\text{Area}(\mathbf{T}) = 4\pi\cdot\pi ab = 4\pi^2 ab$$

47. Compute the surface area of the torus in Exercise 45 using Pappus's Theorem.

SOLUTION The generating curve is the circle of radius a in the (y, z)-plane centered at the point $(0, b, 0)$. The length of the generating curve is $L = \pi a$.



The center of mass of the circle is at the center $(\overline{y}, \overline{z}) = (b, 0)$, and it traverses a circle of radius b centered at the origin. Therefore, the center of mass makes a distance of $2\pi b$. Using Pappus' Theorem, the area of the torus is:

$$L\cdot 2\pi a = 2\pi a\cdot 2\pi b = 4\pi^2 ab.$$

49. Calculate the gravitational potential V for a hemisphere of radius R with uniform mass distribution.

SOLUTION In Exercise 48(b) we expressed the potential V for a sphere of radius R. To find the potential for a hemisphere of radius R, we need only to modify the limits of the angle ϕ to $0 \le \phi \le \frac{\pi}{2}$. This gives the following integral:

$$V(0,0,r) = V(r) = -\frac{Gm}{4\pi}\int_0^{\pi/2}\int_0^{2\pi}\frac{\sin\phi\,d\theta\,d\phi}{\sqrt{R^2+r^2-2Rr\cos\phi}} = -\frac{Gm}{4\pi}\cdot 2\pi\int_0^{\pi/2}\frac{\sin\phi\,d\phi}{\sqrt{R^2+r^2-2Rr\cos\phi}}$$

$$= -\frac{Gm}{4\pi}\int_0^{\pi/2}\frac{\sin\phi\,d\phi}{\sqrt{R^2+r^2-2Rr\cos\phi}}$$

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