

**Solutions Manual**  
**DISCRETE-EVENT SYSTEM SIMULATION**  
*Third Edition*

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33. Let  $X$  be the lifetime of the card in months. The Erlang distribution gives the desired probability where

$$\beta = K = 4, \quad K\theta = 4(1/16) = \frac{1}{4}, \quad \text{and } X = 24$$

Then

$$F(24) = 1 - \sum_{i=0}^3 \frac{e^{-6} 6^i}{i!} = 1 - .151 = .849$$

The complement gives the desired probability, or

$$P(X \geq 2 \text{ years}) = 1 - .849 = .151$$

34. Let  $X$  be defined as the number on a license tag. Then  $X$  is discrete uniform ( $a = 100, b = 999$ ) with cumulative distribution function

$$F(x) = (x - 99)/900, \quad x = 100, 101, \dots, 999$$

(a) The probability that two tag numbers are 500 or higher is

$$[P(X \geq 500)]^2 = [1 - F(499)]^2 = .5556^2 = .3086$$

(b) Let  $Y$  be defined as the sum of two license tag numbers. Then  $Y$  is discrete triangular which can be approximated by

$$F(y) = \begin{cases} (y - a)^2 / [(b - a)(c - a)], & a \leq y \leq b \\ 1 - [(c - y)^2 / [(c - a)(c - b)]], & b \leq y \leq c \end{cases}$$

where  $a = 2(100) = 200$ ,  $c = 2(999) = 1998$ , and  $b = (1998 + 200)/2 = 1099$ .

The probability that the sum of the next two tags is 1000 or higher is

$$P(Y \geq 1000) = 1 - F(999) = .6050$$

35. A normally distributed random variable,  $X$ , with a mean of 10, a variance of 4, and the following properties

$$P(a < X < b) = .90 \text{ and } |\mu - a| = |\mu - b|$$

exists as follows

$$\begin{aligned} P(X < b) = P(X > a) &= .95 \text{ due to symmetry} \\ \Phi[(b - 10)/2] &= .95 \quad b = 13.3 \\ 1 - \Phi[(a - 10)/2] &= .95 \quad a = 6.7 \end{aligned}$$

36. Solution to Exercise 36:

Normal (10, 4)

$$\begin{aligned} F(8) - F(6) &= F\left(\frac{8 - 10}{2}\right) - F\left(\frac{6 - 10}{2}\right) \\ &= F(-1) - F(-2) = (1 - .84134) - (1 - .97725) \\ &= .13591 \end{aligned}$$

$$X = \begin{cases} 1 + \sqrt{27R}, & 0 \leq R \leq 9/27 \\ 10 - \sqrt{54(1-R)}, & 9/27 < R \leq 1 \end{cases}$$

4. Triangular distribution with  $a = 1, c = 10$  and  $E(X) = 4$ . Since  $(a + b + c)/3 = E(X)$ , the mode is at  $b = 1$ . Thus, the height of the triangular pdf is  $h = 2/9$ . (See solution to previous problem. Note that the triangle here is a right triangle.)

Step 1: Find cdf  $F(x)$  = total area from 1 to  $x$ .

$$= 1 - (\text{total area from } x \text{ to } 10).$$

By similar triangles,  $f(x)/h = (10 - x)/(10 - 1)$ , so

$$F(x) = 1 - (10 - x)f(x)/2 = 1 - (10 - x)^2/81, \quad 1 \leq x \leq 10.$$

Step 2: Set  $F(X) = R$  on  $1 \leq X \leq 10$ .

Step 3:  $X = 10 - \sqrt{81(1 - R)}, \quad 0 \leq R \leq 1$

5. Solution to Exercise 5:

$$X = \begin{cases} 6(R - 1/2) & 0 \leq R \leq 1/2 \\ \sqrt{32(R - 1/2)} & 1/2 \leq R \leq 1 \end{cases}$$

6.  $X = 2R^{1/4}, \quad 0 \leq R \leq 1$

7. Solution to Exercise 7:

$$\begin{aligned} F(x) &= x^3/27, \quad 0 \leq x \leq 3 \\ X &= 3R^{1/3}, \quad 0 \leq R \leq 1 \end{aligned}$$

8. Solution to Exercise 8:

Step 1:

$$F(x) = \begin{cases} x/3, & 0 \leq x \leq 2 \\ 2/3 + (x - 2)/24, & 2 < x \leq 10 \end{cases}$$

Step 2: Set  $F(X) = R$  on  $0 \leq X \leq 10$ .

Step 3:

$$X = \begin{cases} 3R, & 0 \leq R \leq 2/3 \\ 2 + 24(R - 2/3) = 24R - 14, & 2/3 < R \leq 1 \end{cases}$$

9. Use Inequality (8.14) to conclude that, for  $R$  given,  $X$  will assume the value  $x$  in  $R_X = \{1, 2, 3, 4\}$  provided

$$F(x - 1) = \frac{(x - 1)x(2x - 1)}{180} < R \leq \frac{x(x + 1)(2x + 1)}{180} = F(x)$$

By direct computation,  $F(1) = 6/180 = .033$ ,  $F(2) = 30/180 = .167$ ,  $F(3) = 42/180 = .233$ ,  $F(4) = 1$ . Thus,  $X$  can be generated by the table look-up procedure using the following table:

$x$	1	2	3	4
$F(x)$	.033	.167	.233	1