Solutions Manual

Discrete-Event System Simulation

Third Edition

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33. Let X be the lifetime of the card in months. The Erlang distribution gives the desired probability where

$$\beta = K = 4,\ K\theta = 4(1/16) = \frac{1}{4},\ \text{and } X = 24$$

Then

$$F(24) = 1 - \sum_{i=o}^{3} \frac{e^6 6^i}{i!} = 1 - .151 = .849$$

The complement gives the desired probability, or

$$P(X \geq 2 \text{ years}) = 1 - .849 = .151$$

34. Let X be defined as the number on a license tag. Then X is discrete uniform $(a = 100, b = 999)$ with cumulative distribution function

$$F(x) = (x - 99)/900, \quad x = 100, 101, \ldots, 999$$

(a) The probability that two tag numbers are 500 or higher is

$$[P(X \geq 500)]^2 = [1 - F(499)]^2 = .5556^2 = .3086$$

(b) Let Y be defined as the sum of two license tag numbers. Then Y is discrete triangular which can be approximated by

$$F(y) = \begin{cases} (y-a)^2/[(b-a)(c-a)], & a \leq y \leq b \\ 1 - [(c-y)^2/[(c-a)(c-b)]], & b \leq y \leq c \end{cases}$$

where $a = 2(100) = 200$, $c = 2(999) = 1998$, and $b = (1998 + 200)/2 = 1099$.

The probability that the sum of the next two tags is 1000 or higher is

$$P(Y \geq 1000) = 1 - F(999) = .6050$$

35. A normally distributed random variable, X, with a mean of 10, a variance of 4, and the following properties

$$P(a < X < b) = .90 \text{ and } |\mu - a| = |\mu - b|$$

exists as follows

$$\begin{aligned} P(X < b) = P(X > a) &= .95 \text{ due to symmetry} \\ \Phi[(b-10)/2] &= .95\ b = 13.3 \\ 1 - \Phi[(a-10)/2] &= .95\ a = 6.7 \end{aligned}$$

36. Solution to Exercise 36:

Normal (10, 4)

$$\begin{aligned} F(8) - F(6) &= F\left(\frac{8-10}{2}\right) - F\left(\frac{6-10}{2}\right) \\ &= F(-1) - F(-2) = (1 - .84134) - (1 - .97725) \\ &= .13591 \end{aligned}$$

$$X = \begin{cases} 1+\sqrt{27R}, & 0 \le R \le 9/27 \\ 10-\sqrt{54(1-R)}, & 9/27 < R \le 1 \end{cases}$$

4. Triangular distribution with $a = 1, c = 10$ and $E(X) = 4$. Since $(a+b+c)/3 = E(X)$, the mode is at $b = 1$. Thus, the height of the triangular pdf is $h = 2/9$. (See solution to previous problem. Note that the triangle here is a right triangle.)

Step 1: Find cdf $F(x) =$ total area from 1 to x.

$$= 1 - \text{ (total area from } x \text{ to } 10).$$

By similar triangles, $f(x)/h = (10-x)/(10-1)$, so

$$F(x) = 1-(10-x)f(x)/2 = 1-(10-x)^2/81, \quad 1 \le x \le 10.$$

Step 2: Set $F(X) = R$ on $1 \le X \le 10$.

Step 3: $X = 10 - \sqrt{81(1-R)}, \quad 0 \le R \le 1$

5. Solution to Exercise 5:

$$X = \begin{cases} 6(R-1/2) & 0 \le R \le 1/2 \\ \sqrt{32(R-1/2)} & 1/2 \le R \le 1 \end{cases}$$

6. $X = 2R^{1/4}, \quad 0 \le R \le 1$

7. Solution to Exercise 7:
$F(x) = x^3/27, \quad 0 \le x \le 3$
$X = 3R^{1/3}, \quad 0 \le R \le 1$

8. Solution to Exercise 8:
Step 1:

$$F(x) = \begin{cases} x/3, & 0 \le x \le 2 \\ 2/3+(x-2)/24, & 2 < x \le 10 \end{cases}$$

Step 2: Set $F(X) = R$ on $0 \le X \le 10$.

Step 3:

$$X = \begin{cases} 3R, & 0 \le R \le 2/3 \\ 2+24(R-2/3) = 24R-14, & 2/3 < R \le 1 \end{cases}$$

9. Use Inequality (8.14) to conclude that, for R given, X will assume the value x in $R_X = \{1,2,3,4\}$ provided

$$F(x-1) = \frac{(x-1)x(2x-1)}{180} < R \le \frac{x(x+1)(2x+1)}{180} = F(x)$$

By direct computation, $F(1) = 6/180 = .033$, $F(2) = 30/180 = .167$, $F(3) = 42/180 = .233$, $F(4) = 1$. Thus, X can be generated by the table look-up procedure using the following table:

x	1	2	3	4
$F(x)$	.033	.167	.233	1