Solutions Manual for Elasticity in Engineering Mechanics, Third Edition

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2-4.6

$$
\begin{aligned}
I_2(\bar{C}) &= \bar{C}_{ij}\bar{C}_{ji} \\
&= a_{i\alpha}a_{j\beta}a_{j\gamma}a_{i\kappa}C_{\alpha\beta}C_{\gamma\kappa} \\
&= a_{\kappa\alpha}a_{\gamma\beta}C_{\alpha\beta}C_{\gamma\kappa} \\
&= C_{\alpha\beta}C_{\alpha\beta} = I_2(C)
\end{aligned}
$$

$$
\begin{aligned}
I_3(\bar{C}) &= \bar{C}_{ij}\bar{C}_{jk}\bar{C}_{ki} \\
&= a_{i\alpha}a_{j\beta}a_{j\gamma}a_{k\kappa}a_{k\chi}a_{i\eta}C_{\alpha\beta}C_{\gamma\kappa}C_{\chi\eta} \\
&= a_{\alpha\eta}a_{\gamma\beta}a_{\chi\kappa}C_{\alpha\beta}C_{\gamma\kappa}C_{\chi\eta} \\
&= C_{\alpha\beta}C_{\beta\chi}C_{\chi\alpha} = I_3(C)
\end{aligned}
$$

2-4.7

By expanding the matrix and finding the determinant, it is straightforward to prove that

$$
I_A = A_{11} + A_{22} + A_{33} = A_{kk}
$$

$$
\begin{aligned}
II_A &= \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} + \begin{vmatrix} A_{11} & A_{13} \\ A_{31} & A_{33} \end{vmatrix} + \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \\
&= \frac{1}{2}\left\{ \left(A_{kk}\right)^2 - \left(A_{ij}A_{ji}\right) \right\}
\end{aligned}
$$

$$
\begin{aligned}
III_A &= \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \\
&= \frac{1}{6}\left\{ 2\left(A_{ij}A_{jk}A_{ki}\right) - 3\left(A_{kk}A_{ij}A_{ji}\right) + \left(A_{kk}\right)^3 \right\}
\end{aligned}
$$

2-15.7 proceeding as in problem (2-15.6), we obtain

$$u = A\left(\frac{x^2}{2} - zx\right) - A\frac{y^2}{2} + c_3 y + c_2 z + c_4$$
$$v = A\left(xy - \frac{y^2}{2}\right) - A\frac{z^2}{2} - c_3 x + c_5 z + c_6$$
$$w = A\left(yz + \frac{z^2}{2}\right) + A\frac{x^2}{2} - c_2 x - c_5 y + c_7$$

For $u = v = w = 0$ at $x = y = z = 0$, $c_4 = c_6 = c_7 = 0$

For $\overline{\omega} = 0$ at $x = y = z = 0$, $c_2 = c_3 = c_5 = 0$

$$\therefore \quad u = A\left(\frac{x^2}{2} - \frac{y^2}{2} - xz\right)$$
$$v = A\left(xy - \frac{y^2}{2} - \frac{z^2}{2}\right)$$
$$w = A\left(\frac{x^2}{2} + yz + \frac{z^2}{2}\right)$$

2-15.8 $u_\alpha = C_{\alpha\beta} x_\beta = C_{\alpha\delta} x_\delta$

$$\epsilon_{\alpha\beta} = \frac{1}{2}\left[u_{\alpha,\beta} + u_{\beta,\alpha} + u_{\theta,\alpha} u_{\theta,\beta}\right]$$

$$\therefore \quad 2\epsilon_{\alpha\beta} = C_{\alpha\beta} + C_{\beta\alpha} + C_{\theta\alpha} C_{\theta\beta}$$

(a) If we take $C_{\alpha\beta} = -C_{\beta\alpha}$, then

$\epsilon_{\alpha\beta} = \frac{1}{2} C_{\theta\alpha} C_{\theta\beta}$ is quadratic in $C_{\alpha\beta}$. Hence the quantities $e_{\alpha\beta}$ vanish.

(b) In case (a) $\epsilon_{\alpha\beta}$ is quadratic in $C_{\alpha\beta}$. Hence, the quantities $e_{\alpha\beta}$ vanish. If we discard quadratic terms in $\epsilon_{\alpha\beta}$, then we discard everything.

2-15.9

Given: $u = -(1 - \cos\phi)\, x - y\sin\phi$

$v = x\sin\phi - (1 - \cos\phi)\, y$

$w = 0 \quad ; \quad \phi = \text{const}$

Then $e_{11} = \frac{\partial u}{\partial x} = -(1 - \cos\phi) \qquad e_{22} = \frac{\partial v}{\partial y} = -(1 - \cos\phi)$

$e_{33} = \frac{\partial w}{\partial z} = 0 \qquad e_{12} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$

$e_{13} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = 0, \quad e_{23} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = 0$

cont'd

3-6.2 In terms of $\sigma_1, \sigma_2, \sigma_3$ (principal stresses)

$$S_x = \sigma_x - \sigma_m = \sigma_1 - \sigma_m = S_1 \;,$$

$$S_y = \sigma_y - \sigma_m = \sigma_2 - \sigma_m = S_2$$

$$S_z = \sigma_z - \sigma_m = \sigma_3 - \sigma_m = S_3$$

$$\sigma_m = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Hence,

$$T_d = \begin{pmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{pmatrix}$$

$$I_{2d} = S_1S_2 + S_1S_3 + S_2S_3 = -\tfrac{1}{6}\left[(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2\right]$$

Since $S_1 + S_2 + S_3 = 0$

$$I_{3d} = S_1 S_2 S_3$$

3-6.3 By Eq (3-3.14) the stress normal to a plane with direction cosine n_α is $\sigma_{nn} = \sigma_{\alpha\beta} n_\alpha n_\beta$. For $(n_1^2, n_2^2, n_3^2) = (\ell^2, m^2, n^2) = (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})$ relative to principal axes $\sigma_{nn} = \sigma_{oct}$. Also, then $\sigma_{11} = \sigma_1$, $\sigma_{22} = \sigma_2$, $\sigma_{33} = \sigma_3$, and $\sigma_{12} = \sigma_{23} = \sigma_{31} = 0$

$$\therefore \sigma_{oct} = \sigma_1 n_1^2 + \sigma_2 n_2^2 + \sigma_3 n_3^2 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \sigma_m$$

Since, $\sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$ is invariant.

4-7.5

(a) simple tension:

By Eqs. (4-7.1), $J_1 = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma}{3\lambda + 2G} = (1-2\nu)\frac{\sigma}{E}$

(b) pure shear:

since $\epsilon_x = \epsilon_y = \epsilon_z = 0$, $J_1 = 0$

Problem Set 4-8

The two problems in this set are quite lengthy, but instructive. The solutions are not presented here, but may be obtained from the authors upon request.

7-10.5 cont'd

term to Eq. (4), V in the form

$$V = \frac{G\theta}{2}\left[\frac{r^2 \cos 2\beta}{\cos\alpha} + a^2 \sum_n A_n \left(\frac{r}{a}\right)^{\lambda} \cos\lambda\beta\right] \tag{5}$$

The first additional term in the bracket of Eq. (5) reduces to $\frac{G\theta r^2}{2}$ for $\beta = \pm\frac{\alpha}{2}$. Therefore, the Σ term in Eq. (5) must vanish for $\beta = \pm\frac{\alpha}{2}$.

Hence, we require

$$\lambda = \frac{n\pi}{\alpha} \quad \text{for } n = 1, 3, 5, \ldots \tag{6}$$

Hence,

$$\phi = \frac{G\theta}{2}\left[-r^2\left(1 - \frac{\cos 2\beta}{\cos\alpha}\right) + a^2 \sum_{n=1,3,5\ldots}^{\infty} A_n \left(\frac{r}{a}\right)^{\frac{n\pi}{\alpha}} \cos\frac{n\pi\beta}{\alpha}\right] \tag{7}$$

To satisfy Eq. (3), Eq. (7) yields

$$\Sigma A_n \cos\frac{n\pi\beta}{\alpha} = 1 - \frac{\cos 2\beta}{\cos\alpha} \tag{8}$$

By the Method of Fourier series (section 7-10), we find

$$A_n = \frac{16\alpha^2}{\pi^3}(-1)^{\frac{n+1}{2}} \frac{1}{n\left(n + \frac{2\alpha}{\pi}\right)\left(n - \frac{2\alpha}{\pi}\right)} \tag{9}$$

Hence, the result given in Problem 7-10.5 is obtained.