Solutions Manual

prepared by
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to accompany

electricity and magnetism

Berkeley Physics Course—Vol. II

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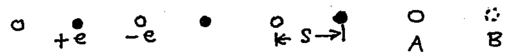
$\boxed{1.8}$

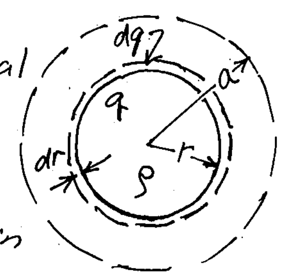


Suppose the array has been built from the left as far as the negative ion A. To add the next ion B, a positive ion, requires an amount of work:

$$-\frac{e^2}{s}+\frac{e^2}{2s}-\frac{e^2}{3s}+\cdots = -\frac{e^2}{s}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right]$$

The expansion of $\ln(1+x)$ is $1-\frac{x}{2}+\frac{x^2}{3}-\cdots$, converging for $-1<x\leq 1$. Evidently the sum in brackets above is just $\ln 2$, or 0.693. Thus the energy of the chain, per ion, is $-0.693\frac{e^2}{s}$

$\boxed{1.9}$ The charge in the sphere of radius r is: $q=\frac{4\pi}{3}r^3\rho$. Its external field is the same as if q were at the center. The next shell to be added contains charge dq: $dq=4\pi r^2 dr\,\rho$. The work done in bringing up dq is therefore:



$$dW=\frac{q\,dq}{r}=\frac{(4\pi\rho)^2}{3}r^4\,dr.$$

Building up the whole sphere this way requires the work:

$$W=\int_0^a\frac{(4\pi\rho)^2}{3}r^4\,dr=\frac{(4\pi\rho)^2}{3}\frac{a^5}{5}$$

The charge in the complete sphere is: $Q=\frac{4\pi}{3}a^3\rho$. Thus the potential energy U, which is the same as W, can be written: $U=\frac{3}{5}Q^2/a$. Notice that Q^2/a has the proper dimensions (charge)^2/distance. Indeed we could have predicted that much of our result without calculation.

$\boxed{1.10}$ Setting the potential energy $\frac{3}{5}\frac{e^2}{r_0}$ equal to mc^2 and solving for r_0:

$$r_0=\frac{3}{5}\frac{e^2}{mc^2}=\frac{3}{5}\frac{(4.80\times10^{-10})^2}{(.91\times10^{-27})(3\times10^{10})^2}=1.69\times10^{-13}\text{ cm}$$

$\boxed{1.11}$

ρ_0 $Q=92e$ radius a $U=\frac{3}{5}\frac{Q^2}{a}$

ρ_0 $Q'=\frac{Q}{2}$ ρ_0 $Q'=\frac{Q}{2}$ radius $a'=\frac{a}{2^{1/3}}$ $U'=\frac{3}{5}\frac{Q'^2}{a'}$

$$U'=\frac{3}{5}\frac{(Q/2)^2}{(a/2^{1/3})}=\frac{3}{5}\frac{Q^2}{a}\cdot\frac{1}{2^{5/3}}$$

The decrease in energy is $U-2U'$

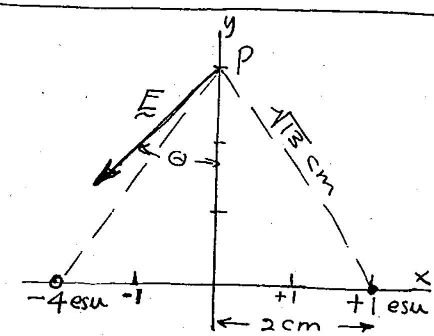
$$U-2U'=\frac{3}{5}\frac{Q^2}{a}\left[1-\frac{2}{2^{5/3}}\right]=\frac{3}{5}\frac{Q^2}{a}(.370)=.222\frac{Q^2}{a}$$

With ρ_0 given, a is determined by $\frac{4\pi}{3}a^3\rho_0=Q$, from which $Q^2/a=\left(\frac{4\pi\rho_0}{3}\right)^{1/3}Q^{5/3}$ Thus:

$$U-2U'=.222\left[\frac{4\pi}{3}\cdot4\times10^{28}\right]^{\frac{1}{3}}\left[92\times4.8\times10^{-10}\right]^{5/3}=6.7\times10^{-4}\text{ ergs}$$

$1\text{ Mev}=1.6\times10^{-6}$ ergs. Hence $U-2U'=420$ Mev.

$\boxed{1.12}$ (a) At the point P,



$$E_x=-\frac{1}{(\sqrt{13})^2}\cdot\frac{2}{\sqrt{13}}-\frac{4}{(\sqrt{13})^2}\cdot\frac{2}{\sqrt{13}}=-\frac{10}{13^{3/2}}\text{ statvolts/cm}$$

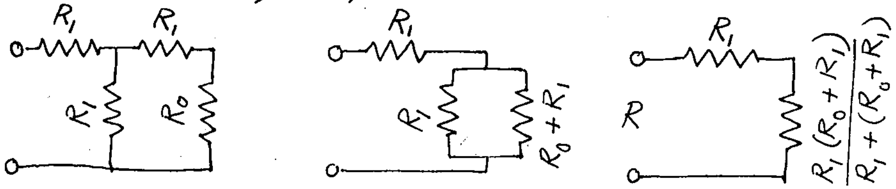
$$E_y=\frac{1}{(\sqrt{13})^2}\cdot\frac{3}{\sqrt{13}}-\frac{4}{(\sqrt{13})^2}\cdot\frac{3}{\sqrt{13}}=\frac{-9}{13^{3/2}}\text{ statvolts/cm}$$

$$E=\left(E_x^2+E_y^2\right)^{\frac{1}{2}}=\left(\frac{100+81}{13^3}\right)^{\frac{1}{2}}=(.0825)^{\frac{1}{2}}=0.288$$

$E=0.288$ statvolts/cm. $\theta=\tan^{-1}\frac{10}{9}=48°$

(b) A point where the field is zero must lie on the x-axis, nearer the smaller of the two charges, but not between them. The condition is $\frac{1}{(x-2)^2}=\frac{4}{(x+2)^2}$. This has two roots $x=6$ and $x=\frac{2}{3}$. At $x=\frac{2}{3}$ the fields of the two sources are equal but not opposite. The only point where the field is zero is the point $x=6$, $y=0$, $z=0$.

4.11 To find the input resistance R, reduce the circuit step by step:

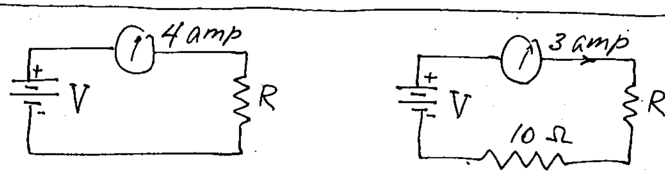


$R = R_1 + \dfrac{R_1(R_0+R_1)}{R_1+(R_0+R_1)}$ We want $R = R_0$, or

$R_0(2R_1+R_0) = R_1(2R_1+R_0) + R_1(R_1+R_0)$.

Collecting terms: $R_0^2 = 3R_1^2$ or $R_1 = R_0/\sqrt{3}$

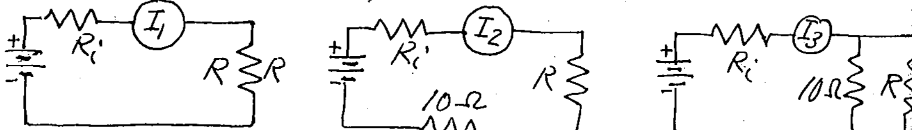
4.12



$V = 4R$ $V = 3(R+10)$

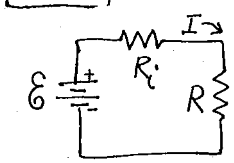
Solve for R and V: $R = 30\ \Omega$ $V = 120$ volts

If the battery had internal resistance R_i, three measurements of current would be needed to determine R_i, R, and V, for instance:



$V = I_1(R_i+R)$ $V = I_2(R_i+R+10)$ $V = I_3\left(R_i + \dfrac{10R}{R+10}\right)$

4.13 Let P = power dissipated in resistor R

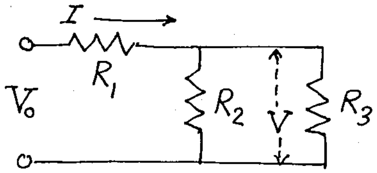


$P = I^2R$ $I = \dfrac{\mathcal{E}}{R+R_i}$ $P = \dfrac{\mathcal{E}^2R}{(R+R_i)^2}$

$\dfrac{dP}{dR} = \mathcal{E}^2\,\dfrac{(R+R_i)^2 - 2R(R+R_i)}{(R+R_i)^4} = \mathcal{E}^2\,\dfrac{R_i-R}{(R+R_i)^3}$

For $R = R_i$ $\dfrac{dP}{dR} = 0$. Also, $\dfrac{dP}{dR} < 0$ for $R > R_i$, and $\dfrac{dP}{dR} > 0$ for $R < R_i$. Hence this is condition for maximum P.

4.14 Let P be the power dissipated in resistor R_3.



$I = \dfrac{V_0}{R_1 + \dfrac{R_2R_3}{R_2+R_3}}$

$= \dfrac{V_0(R_2+R_3)}{R_1R_2 + R_1R_3 + R_2R_3}$

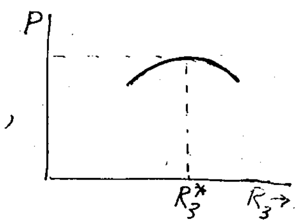
$V = V_0 - IR_1 = V_0 - \dfrac{V_0R_1(R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3} = V_0\,\dfrac{R_2R_3}{R_1R_2+R_1R_3+R_2R_3}$

$P = \dfrac{V^2}{R} = V_0^2\,\dfrac{R_2^2R_3}{(R_1R_2+R_1R_3+R_2R_3)^2}$. We want $\dfrac{\partial P}{\partial R_3} = 0$:

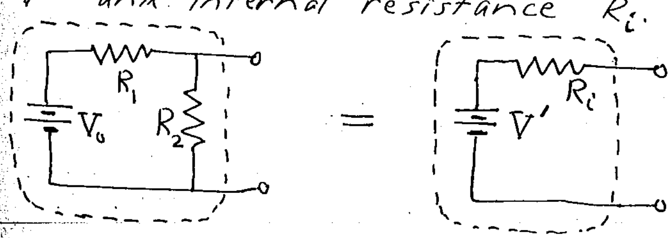
$R_2^2(R_1R_2+R_1R_3+R_2R_3)^2 - 2R_2^2R_3(R_1+R_2)(R_1R_2+R_1R_3+R_2R_3) = 0$

This reduces to $R_1R_2 - R_2R_3 - R_1R_3 = 0$, or $R_3 = \dfrac{R_1R_2}{R_1+R_2}$

If this condition is satisfied, P is an extremum (actually a maximum) with respect to variation of R_3 with R_1 and R_2 constant. Denote by R_3^* this value of R_3. A small change in R_3, $R_3 \to R_3^* + \Delta R_3$, will cause only a second-order change in P, that is, $\Delta P \sim (\Delta R_3)^2$.



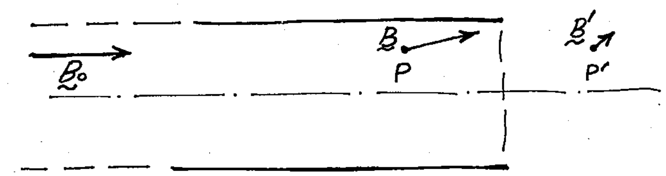
Comment: Note that $\dfrac{R_1R_2}{R_1+R_2}$ is just the resistance of R_1 and R_2 in parallel. As "seen by" R_3 the constant voltage source V_0 and the resistors R_1 and R_2 are equivalent to a battery with emf V' and internal resistance R_i.



The open-circuit voltage of the box on the left is the end face, only the axial field component is involved. Therefore the flux must be just half the interior flux.

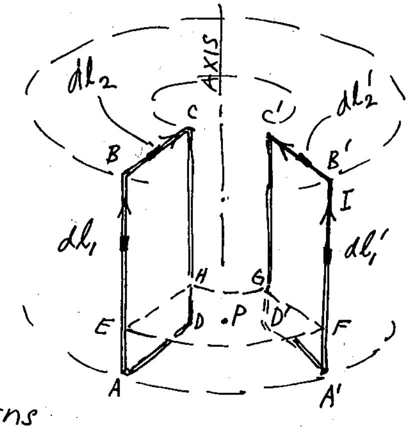
(c) For the same reason, the flux tube on the surface of which the field line CDE lies must flare out as it approaches the end face so that its cross-section there becomes a circle of twice the area, containing the same amount of flux. $\pi r^2 = 2\pi r_0^2$, or $r = \sqrt{2}\,r_0$

The arguments used in (b) and (c) lead to a more general statement about the field of the semi-infinite solenoid:

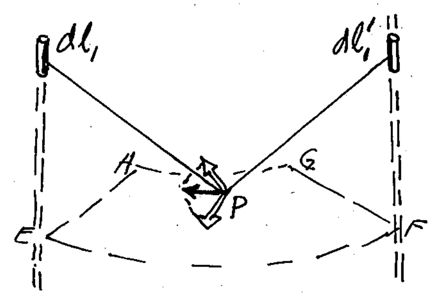


At two corresponding points P and P′, symmetrically located with respect to the end plane and equidistant from the axis, the fields $\mathbf{B}$ and $\mathbf{B}'$ are related as follows: The radial components of $\mathbf{B}$ and $\mathbf{B}'$ are equal. The sum of the axial components of $\mathbf{B}$ and $\mathbf{B}'$ is equal to B_0, if P lies inside the coil, or to zero, if P lies outside the coil. The conclusion of (b) and (c) follow in the special case in which P and P′ coincide.

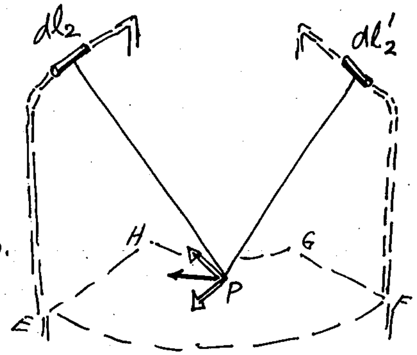
$\boxed{6.19}$ Consider any point such as P. We want to show that the field $\mathbf{B}$ at P is "circumferential", that is, it lies in the plane EFGH, a plane containing P and perpendicular to the axis, and it is perpendicular to the radius from P to the axis. Consider a pair of turns ABCD and A′B′C′D′ symmetrically located with respect to P.



(1) The contributions to the field at P from equal, symmetrically located wire-elements $d\ell_1$ and $d\ell_1'$ combine to form a resultant in the circumferential direction. (Note that each vector lies in the plane EFGH.)

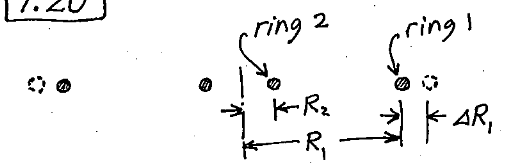


(2) The contributions to the field at P from equal, symmetrically located wire-elements $d\ell_2$ and $d\ell_2'$ also combine to form a resultant in the circumferential direction. Here the individual vectors are out of the plane, and the geometry is a little harder to visualize.



Since the two rectangular turns can be entirely decomposed into such elements, and the entire coil can be decomposed into such pairs of turns, it follows that the total field at P must be circumferential.

7.20



With current I in the outer ring the flux through the inner ring, for $R_1 \gg R_2$, is:

$$\Phi_{21} = \pi R_2^2 \cdot \frac{2\pi I}{cR_1}$$

or $\Phi_{21} = \dfrac{2\pi^2 I}{c}\dfrac{R_2^2}{R_1}$. Suppose we change R_1 to $R_1 + \Delta R_1$, by expanding the outer ring while holding I constant. The resulting change in Φ_{21} is:

$$\Delta\Phi_{21} = \frac{\partial \Phi_{21}}{\partial R_1}\Delta R_1 = -\frac{2\pi^2 I}{c}\frac{R_2^2}{R_1^2}\Delta R_1$$

Now consider a current I in the inner ring, ring 2. Let B be the field strength at the radius of the outer ring, R_1. If we now expand the outer ring by ΔR_1 the flux Φ_{12} decreases by just the amount of flux between the circle of radius R_1 and the circle of radius $R_1 + \Delta R_1$. (Problem 7.19 explained why it is a decrease.) The change in flux is

$$\Delta\Phi_{21} = -B\cdot 2\pi R_1 \Delta R_1$$

since $2\pi R_1 \Delta R_1$ is the area between the circles.

Our theorem $\Phi_{12} = \Phi_{21}$ guarantees that $\Delta\Phi_{12} = \Delta\Phi_{21}$

Hence:

$$-\frac{2\pi^2 I}{c}\frac{R_2^2}{R_1^2}\Delta R_1 = -B\cdot 2\pi R_1 \Delta R_1$$

Solving for B: $\quad B = \dfrac{\pi R_2^2 I}{cR_1^3}$ or more generally, $B = \dfrac{\pi R_2^2 I}{cr^3}$ at any point in the plane of the ring where $r \gg R_2$.

7.21 Assume current I flows in the outer solenoid. The field inside, approximately uniform in the region occupied by the inner solenoid, is $B = \dfrac{4\pi I}{c}\dfrac{N_2}{b_2}$.

We have assumed here that $\dfrac{b_2}{a_2}$ is so large that we can use the formula for an infinite solenoid. We can refine this by using Eq. 6.44, page 203, to calculate the field at the center of a finite solenoid of length b_2, radius a_2. The correction factor is simply $\cos\left(\tan^{-1}\dfrac{2a_2}{b_2}\right)$ or $b_2/\sqrt{b_2^2+4a_2^2}$. This will still not lead us to an exact result, for the inner coil includes a finite volume in which the field strength varies somewhat. But for the proportions shown in the figure, the approximation will be pretty good.

The flux linking the inner coil is

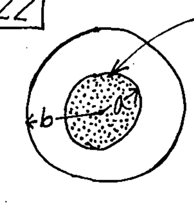
$$\Phi_{12} = \pi a_1^2 B N_1 \quad\text{or}\quad \frac{4\pi^2 I}{c}\frac{N_1 N_2 a_1^2}{b_2}\left(\frac{b_2}{\sqrt{b_2^2+4a_2^2}}\right)$$

Since $\mathcal{E}_{12} = -\dfrac{1}{c}\dfrac{d\Phi_{12}}{dt} = -M\dfrac{dI}{dt}$, we have, in CGS units, $M = \dfrac{4\pi^2 N_1 N_2 a_1^2}{c^2\, b_2}\left(\dfrac{b_2}{\sqrt{b_2^2+4a_2^2}}\right)$

To express M in henrys, replace the constant $\dfrac{1}{c^2}$ by 10^{-9}:

$$M = \frac{4\pi^2\times 10^{-9} N_1 N_2 a_1^2}{\sqrt{b_2^2+4a_2^2}} \text{ henrys}$$

7.22



Current I is uniformly distributed over the cross-section of the inner conductor, and returns in outer conductor whose thickness we neglect.

For $r<a$, $\dfrac{4\pi I}{c}\dfrac{r^2}{a^2} = 2\pi r B$, or $B = \dfrac{2Ir}{ca^2}$

For $a<r<b$, $B = \dfrac{2I}{cr}$ $\quad$ For $r>b$, $B=0$

$$\text{Energy stored} = \frac{1}{8\pi}\int B^2 dv = \frac{\ell}{8\pi}\int_0^a \left(\frac{2Ir}{ca^2}\right)^2 2\pi r\,dr + \frac{\ell}{8\pi}\int_a^b \left(\frac{2I}{cr}\right)^2 2\pi r\,dr$$

$$= \frac{\ell I^2}{c^2 a^4}\int_0^a r^3 dr + \frac{\ell I^2}{c^2}\int_a^b \frac{dr}{r} = \frac{\ell I^2}{c^2}\left(\frac{1}{4} + \ln\frac{b}{a}\right) = \frac{1}{2}LI^2$$

Hence $L = \dfrac{2\ell}{c^2}\left(\dfrac{1}{4} + \ln\dfrac{b}{a}\right)$. If the current I is changing very rapidly, the current density will not be uniform in the inner conductor. In the high-frequency limit, the current is all on the surface at $r=a$, and $L \to \dfrac{2\ell}{c^2}\ln\dfrac{b}{a}$.

or 2.85 × 10 gms. The ratio of the absolute temperatures is $\frac{273}{90}$. Thus the volume susceptibility of air at NTP should be

$$2.45\times10^{-4}\left(\frac{90}{273}\right)\left(2.85\times10^{-4}\right) = 2.30\times10^{-8}$$

10.3 Eq. 6.41, p. 202: $B_z = \dfrac{2\pi b^2 I}{c(b^2+z^2)^{3/2}}$

For $z \gg b$, we can neglect b^2 compared to z^2 in the denominator, and the formula reduces to

$$B_z \approx \frac{2\pi b^2 I}{c z^3}$$

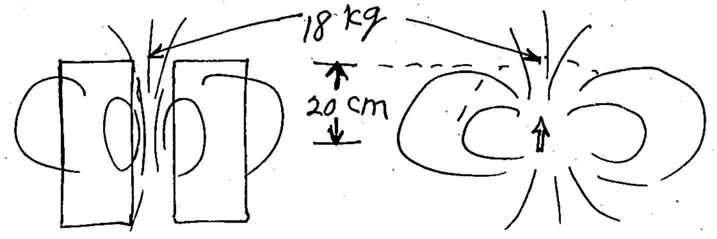
Since πb^2 is the area of the loop, its magnetic moment m is $\frac{\pi b^2 I}{c}$, and the expression above can be written $B_z \approx \frac{2m}{z^3}$, which is just the field of a dipole on its axis.

More exactly, the factor $(b^2+z^2)^{-3/2}$ can be expanded in powers of $\left(\frac{b}{z}\right)^2$:

$$B = \frac{2\pi b^2 I}{c z^3}\left(1+\frac{b^2}{z^2}\right)^{-3/2} = \frac{2\pi b^2 I}{c z^3}\left(1-\frac{3}{2}\frac{b^2}{z^2}+\ldots\right)$$

from which we see that the field will be within 1% of the dipole field if $\frac{3}{2}\frac{b^2}{z^2} < \frac{1}{100}$, or $z > \sqrt{150\, b^2} \approx 12\, b$.

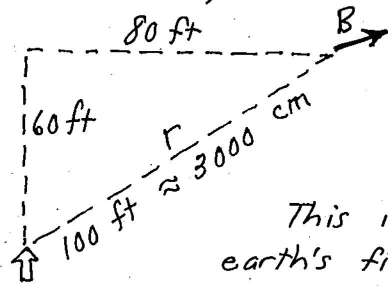
10.4



To estimate roughly the magnetic dipole moment

of the solenoid, let us suppose that it is equivalent to a point dipole which would produce, 20 cm away on its axis, a field strength B_z equal to that at the end of the solenoid, namely 18000 gauss. This is reasonable because the magnetic field configuration near the end of the solenoid and beyond looks not very different from a dipole field. On this assumption,

$$18000 = \frac{2m}{(20)^3}, \quad \text{or} \quad m = 7.2\times10^{7} \text{ cgs units } *$$



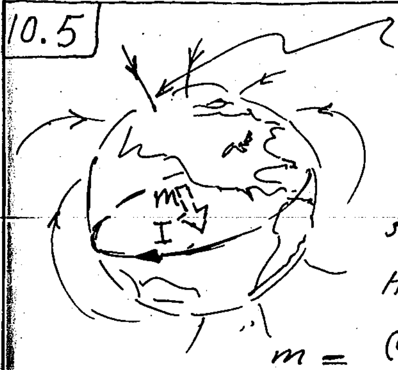
In order of magnitude,

$$B \approx \frac{m}{r^3} = \frac{7\times10^7}{27\times10^9} = 2.5\times10^{-3}\text{ gauss}$$

This is small compared to the earth's field, and if it were perfectly steady could not be noticed. But if frequently switched on and off it might cause trouble.

* An exact calculation, taking into account the actual current distribution in the coil, gives for its dipole moment the value 6.2×10^7, so the above estimate is pretty good.

10.5



North Magnetic pole

$B = 0.62$ gauss

$r = 6400$ km $= 6.4\times10^8$ cm

For central dipole of strength m, $B = \frac{2m}{r^3}$

Hence $m = \frac{Br^3}{2}$

$$m = \frac{(0.62)(6.4\times10^8)^3}{2} = 8\times10^{25}\text{ cgs units}$$

Current I amps flowing in ring of radius r has dipole moment $m = \frac{\pi I r^2}{10}$. Hence the equatorial current required to make a dipole moment