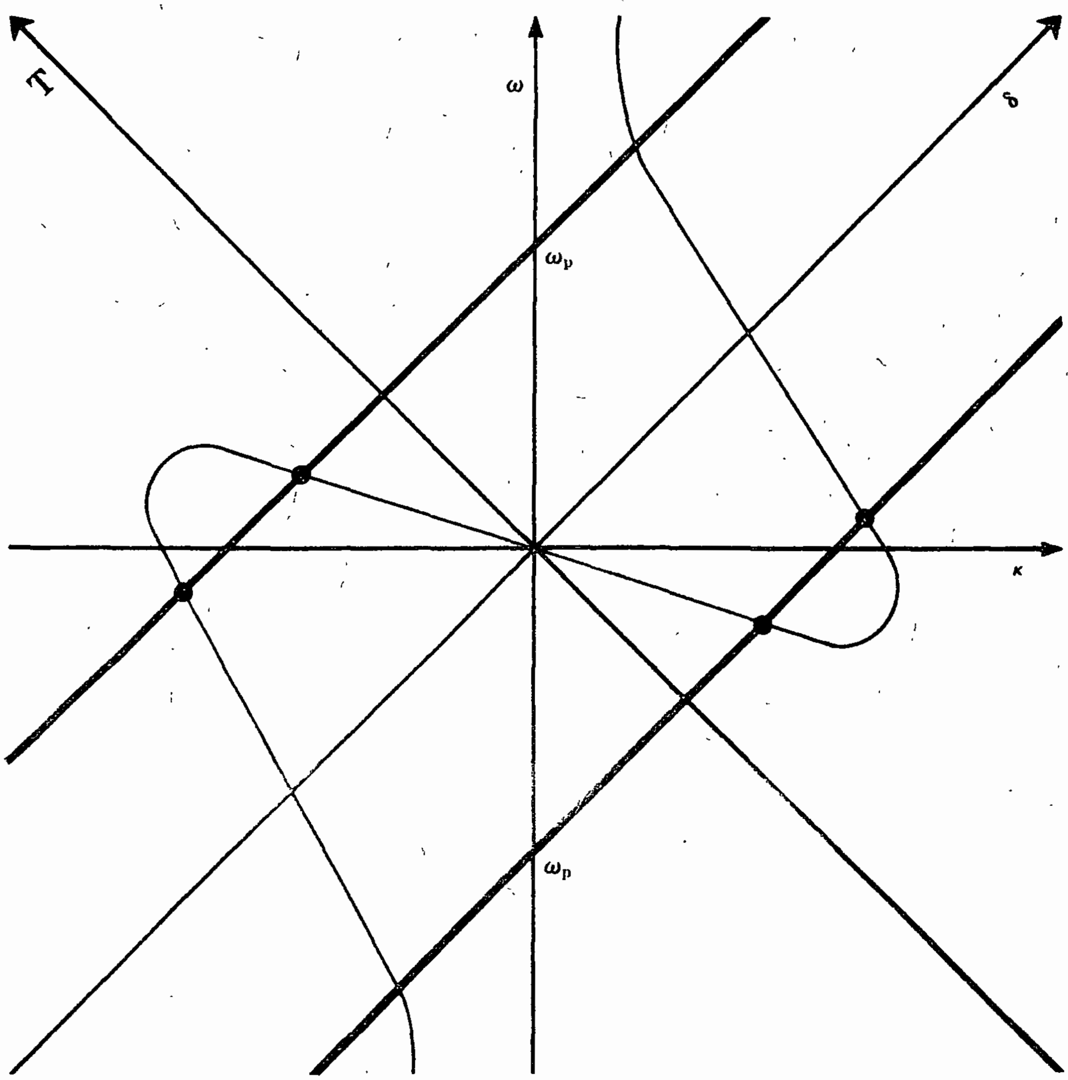
SOLUTIONS MANUAL FOR

ELECTROMECHANICAL DYNAMICS

PART III: Elastic and Fluid Media

HERBERT H. WOODSON JAMES R. MELCHER



Prepared by MARKUS ZAHN

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Part III: Elastic and Fluid Media

HERBERT H. WOODSON
Philip Sporn Professor of Energy Processing
Departments of Electrical and Mechanical Engineering

JAMES R. MELCHER
Associate Professor of Electrical Engineering
Department of Electrical Engineering

both of Massachusetts Institute of Technology

Prepared by

MARKUS ZAHN
Massachusetts Institute of Technology

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PROBLEM 12.14

Part a

The force equation in the y direction is

$$\frac{\partial p}{\partial y} = -\rho g \tag{a}$$

Thus

$$p = -\rho g(y-\xi) \tag{b}$$

where we have used the fact that at $y=\xi$, the pressure is zero.

Part b

$\nabla\cdot\bar{v} = 0$ implies

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{c}$$

Integrating with respect to y, we obtain

$$v_y = -\frac{\partial v_x}{\partial x} y + C \tag{d}$$

where C is a constant of integration to be evaluated by the boundary condition at $y = -a$, that

$$v_y(y=-a) = 0$$

since we have a rigid bottom at $y = -a$.

Thus

$$v_y = -\frac{\partial v_x}{\partial x}(y+a) \tag{e}$$

Part c

The x-component of the force equation is

$$\rho\frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} = -\rho g\frac{\partial \xi}{\partial x} \tag{f}$$

or

$$\frac{\partial v_x}{\partial t} = -g\frac{\partial \xi}{\partial x} \tag{g}$$

Part d

At $y = \xi$,

$$v_y = \frac{\partial \xi}{\partial t} \tag{h}$$

Thus, from part (b), at $y = \xi$

$$\frac{\partial \xi}{\partial t} = -\frac{\partial v_x}{\partial x}(\xi+a) \tag{i}$$

However, since $\xi \ll a$, and v_x and v_y are small perturbation quantities, we can approximately write

$$\frac{\partial \xi}{\partial t} = -a\frac{\partial v_x}{\partial x} \tag{j}$$

Part e

Our equations of motion are now

PROBLEM 12.28

Part a

The magnetic field within the generator is

$$\bar{B} = \frac{\mu_o N i}{w} \bar{i}_2 \tag{a}$$

The current through the generator is

$$\bar{J} = \bar{i}_3 \frac{i}{\ell w} = \sigma\left(\frac{v}{D} + VB\right)\bar{i}_3 \tag{b}$$

Solving for v, the voltage across the channel, we obtain

$$v = \left(\frac{D}{\sigma \ell w} - \frac{V\mu_o N}{w} D\right) i \tag{c}$$

We apply Faraday's law around the electrical circuit to obtain

$$v + \frac{1}{C}\int i dt + i R_L = -\frac{\mu_o N^2}{w} \ell d \frac{di}{dt} \tag{d}$$

Differentiating and simplifying this equation we finally obtain

$$\frac{d^2 i}{dt^2} + \left(\frac{R_L w}{\mu_o N^2 \ell d} + \frac{D}{\sigma L w} - \frac{\mu_o NDV}{w}\right)\frac{di}{dt} + \frac{w}{\mu_o N^2 \ell d C} i = 0 \tag{e}$$

We assume that $i = \text{Re}\, \hat{I} e^{st}$.

Substituting this assumed solution back into the differential equation, we obtain

$$s^2 + \left(\frac{R_L w}{\mu_o N^2 \ell d} + \frac{D}{\sigma L w} - \frac{\mu_o NDV}{w}\right) s + \frac{w}{\mu_o N^2 \ell d C} = 0 \tag{f}$$

Solving, we have

$$s = -\frac{\left(\frac{R_L w}{\mu_o N^2 \ell d} + \frac{D}{\sigma L w} - \frac{\mu_o NDV}{w}\right)}{2} \pm \sqrt{\frac{\left(\frac{R_L w}{\mu_o N^2 \ell d} + \frac{D}{\sigma L w} - \frac{\mu_o NDV}{w}\right)^2}{4} - \frac{w}{\mu_o N^2 \ell d C}} \tag{g}$$

For the device to be a pure ac generator, we must have that s is purely imaginary, or

$$R_L = \left(\frac{\mu_o NDV}{w} - \frac{D}{\sigma L w}\right) \frac{\mu_o N^2 \ell d}{w} \tag{h}$$

Part b

The frequency of operation is then

$$\omega = \frac{w}{\mu_o N^2 \ell d C} \tag{i}$$

PROBLEM 12.29

Part a

The current within the MHD generator is

$$\bar{J} = \frac{i}{\ell d} \bar{i}_y = \sigma\left(\frac{V}{w} + vB_o\right)\bar{i}_y \tag{a}$$

PROBLEM 12.31 (continued)

Solving for v we obtain

$$v = Ae^{-t/\tau} + \frac{\Delta p_o}{\left(\dfrac{\Delta p_o}{v_o} + \dfrac{B_o w}{R_L + R_i}\right)} \qquad \text{where } R_i = \frac{w}{\sigma \ell d} \tag{h}$$

and where

$$\tau = \frac{\rho\,\ell}{\left[\dfrac{\Delta p_o}{v_o} + \dfrac{wB_o}{R_L + R_i}\right]} \tag{i}$$

at t = 0, the velocity must be continuous. Therefore,

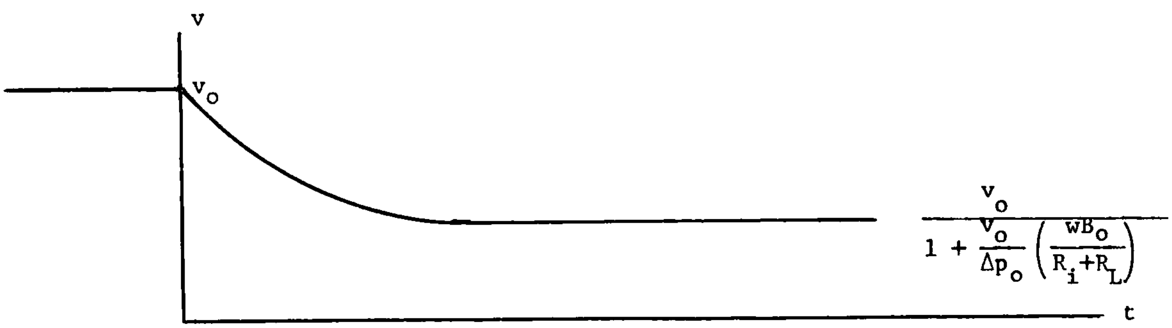
$$A = v_o - \frac{\Delta p_o}{\left(\dfrac{\Delta p_o}{v_o} + \dfrac{w B_o}{R_L + R_i}\right)}$$

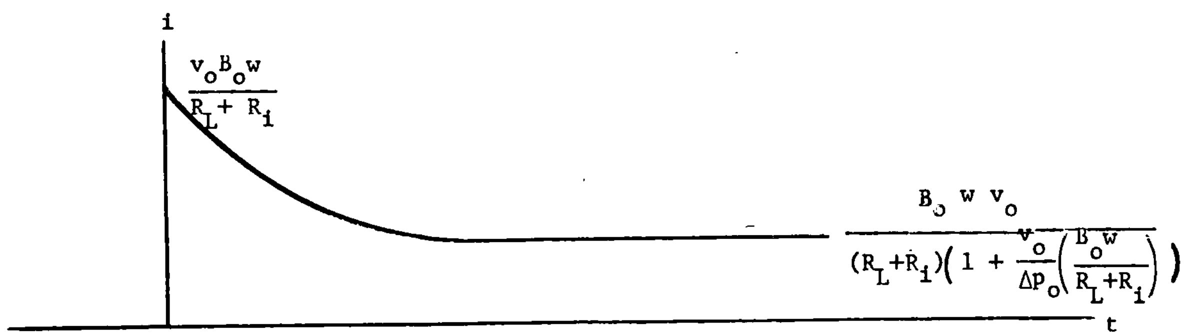
Now, the current is

$$i = \frac{wB_o\, v}{R_L + R_i} \tag{k}$$

Thus

$$i = \left(\frac{wB_o}{R_L \;\; R_i}\right)\left[\frac{\Delta p_o}{\left(\dfrac{\Delta p_o}{v_o} + \dfrac{wB_o}{R_L + R_i}\right)}(1 - e^{-t/\tau}) + v_o e^{-t/\tau}\right] \tag{ℓ}$$





PROBLEM 13.11 (continued)

We replace $\partial/\partial t$ by $\partial/\partial t + v\cdot\nabla$ to obtain

$$\left(\frac{\partial}{\partial t} + V_o \frac{\partial}{\partial x}\right)^2 v' = a_s^2 \frac{\partial^2 v'}{\partial x^2}$$

Letting $v' = \text{Re}\, \hat{V} e^{j(\omega t - kx)}$

we have

$$(\omega - kV_o)^2 = a_s^2 k^2$$

Solving for w, we obtain

$$\omega = k(V_o \pm a_s)$$

Part b

Solving for k, we have

$$k = \frac{\omega}{V_o \pm a_s}$$

For $V_o > a_s$, both waves propagate in the positive x- direction.

PROBLEM 13.12

Part a

We assume that

$$\bar{E} = \bar{i}_z E_z(x,t)$$
$$\bar{J} = \bar{i}_z J_z(x,t)$$
$$\bar{B} = \bar{i}_y \mu_o[H_o + H_y'(x,t)]$$

We also assume that all quantities can be written in the form of Eq. (13.2.91) .

$$\rho_o \frac{\partial v_x}{\partial t} = -\frac{\partial p'}{\partial x} - J_z\mu_o H_o \quad \text{(conservation of momentum linearized)} \tag{a}$$

The relevant electromagnetic equations are

$$\frac{\partial H_y'}{\partial x} = J_z \tag{b}$$

and

$$\frac{\partial E_z}{\partial x} = \mu_o \frac{\partial H_y'}{\partial t} \tag{c}$$

and the constitutive law is

$$J_z = \sigma(E_z + v_x\mu_o H_o) \tag{d}$$

We recognize that Eqs. (13.2.94), (13.2.96) and (13.2.97) are still valid, so

$$\frac{1}{\rho_o}\frac{\partial \rho'}{\partial t} = -\frac{\partial v_x}{\partial x} \tag{e}$$