An Instructor's Solutions Manual to Accompany

Fundamentals of Digital Signal Processing using MATLAB, 2nd Edition

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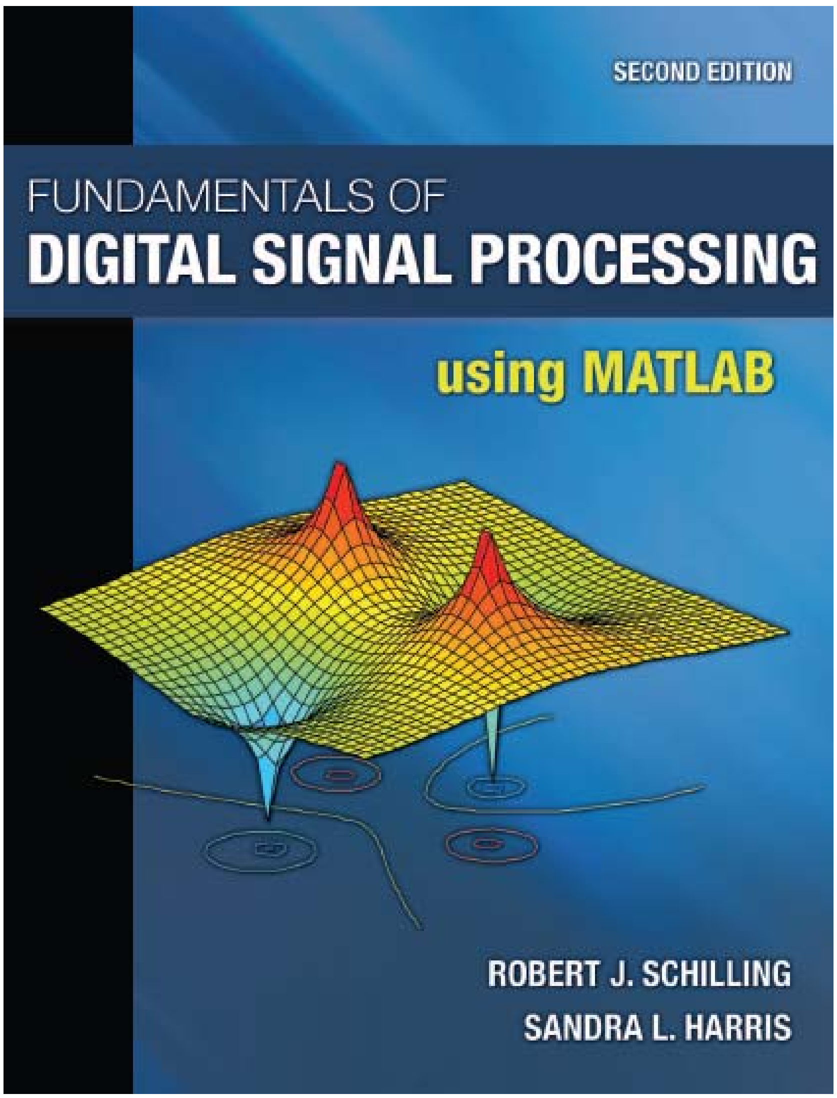

CENGAGE Learning

Contents

$\boxed{2.32}$ Consider a linear discrete-time system S with input x and output y. Suppose S is driven by an input $x(k)$ for $0 \le k < L$ to produce a zero-state output $y(k)$. Use deconvolution to find the impulse response $h(k)$ for $0 \le k < L$ if $x(k)$ and $y(k)$ are as follows.

$$\begin{aligned} x &= [2, 0, -1, 4]^T \\ y &= [6, 1, -4, 3]^T \end{aligned}$$

Solution

Using (2.7.15) and Example 2.16 as a guide

$$\begin{aligned} h(0) &= \frac{y(0)}{x(0)} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

Applying (2.7.18) with $k = 1$ yields

$$\begin{aligned} h(1) &= \frac{y(1) - h(0)x(1)}{x(0)} \\ &= \frac{1 - 3(0)}{2} \\ &= .5 \end{aligned}$$

Applying (2.7.18) with $k = 2$ yields

$$\begin{aligned} h(2) &= \frac{y(2) - h(0)x(2) - h(1)x(1)}{x(0)} \\ &= \frac{-4 - 3(-1) - .5(0)}{2} \\ &= -.5 \end{aligned}$$

Finally, applying (2.7.18) with $k = 3$ yields

$$\begin{aligned} h(3) &= \frac{y(3) - h(0)x(3) - h(1)x(2) - h(2)x(1)}{x(0)} \\ &= \frac{3 - 3(4) - .5(-1) + .5(0)}{2} \\ &= -4.25 \end{aligned}$$

$\boxed{3.31}$ Consider a running average filter of order $M-1$.

$$y(k) = \frac{1}{M}\sum_{i=0}^{M-1} x(k-i)$$

(a) Find the transfer function $H(z)$. Express it as a ratio of two polynomials in z.

(b) Use the geometric series in (3.2.3) to show that an alternative form of the transfer function is as follows. *Hint*: Express $y(k)$ as a difference of two sums.

$$H(z) = \frac{z^M - 1}{M(z-1)z^{M-1}}$$

(c) Convert the transfer function in part (b) to a difference equation.

Solution

(a) Using the delay property

$$Y(z) = \frac{1}{M}\sum_{i=0}^{M-1} z^{-i} X(z)$$

Thus the transfer function is

$$\begin{aligned} H(z) &= \frac{Y(z}{X(z)} \\ &= \frac{1}{M}\sum_{i=0}^{M-1} z^{-i} \\ &= \frac{1 + z^{-1} + \cdots + z^{-M+1}}{M} \\ &= \frac{z^{M-1} + z^{M-2} + \cdots + 1}{Mz^{M-1}} \end{aligned}$$

(b) Starting with the hint

$$\begin{aligned} y(k) &= \frac{1}{M}\sum_{i=0}^{M-1} x(k-i) \\ &= \frac{1}{M}\left[\sum_{i=0}^{\infty} x(k-i) - \sum_{i=M}^{\infty} x(k-i)\right] \end{aligned}$$

$\boxed{4.18}$ Let $x(k)$ be an N-point signal. Starting with the definition of average power in (4.3.40), use Parseval's identity to show that the average power is the average of the power density spectrum.

Solution

Using the definition of average power and Parseval's identity

$$
\begin{aligned}
P_x &\triangleq \frac{1}{N}\sum_{i=0}^{N-1}|x(k)|^2 \\
&= \frac{1}{N}\left[\frac{1}{N}\sum_{i=0}^{N-1}|X(i)|^2\right] \\
&= \frac{1}{N}\left[\sum_{i=0}^{N-1}\frac{|X(i)|^2}{N}\right] \\
&= \frac{1}{N}\sum_{i=0}^{N-1}S_x(i)
\end{aligned}
$$

$\boxed{5.7}$ Consider a type 1 FIR linear-phase filter of order $m = 2$ with coefficient vector $b = [1, 1, 1]^T$.

(a) Find the transfer function, $H(z)$.

(b) Find the amplitude response, $A_r(f)$.

(c) Find the zeros of $H(z)$.

Solution

(a) Using Example 5.3 as a guide,

$$H(z) = 1 + z^{-1} + z^{-2}$$

(b) Let $\theta = 2\pi fT$. Using Euler's identity, the frequency response is

$$\begin{aligned} H(f) &= H(z)|_{z=\exp(j\theta)} \\ &= 1 + \exp(-j\theta) + \exp(-j2\theta) \\ &= \exp(-j\theta)[\exp(j\theta) + 1 + \exp(-j\theta)] \\ &= \exp(-j\theta)[1 + 2\text{Re}\{\exp(j\theta)\}] \\ &= \exp(-j\theta)[1 + 2\cos(\theta)] \\ &= \exp(-j2\pi f)A_r(f) \end{aligned}$$

Thus the amplitude response is

$$A_r(f) = 1 + 2\cos(2\pi fT)$$

(c) The numerator of $H(z)$ is $b(z) = z^2 + z + 1$. Thus the zeros of $H(z)$ are

$$\begin{aligned} z_{1,2} &= \frac{-1 \pm \sqrt{-3}}{2} \\ &= \frac{-1 \pm j\sqrt{3}}{2} \end{aligned}$$

$\boxed{7.14}$ Design a second-order analog lowpass Chebyshev-I filter, $H_a(s)$, using $F_p = 10$ Hz and $\delta_p = .1$.

Solution

First one must locate the poles. Using (7.4.17), the ripple factor parameter is

$$\begin{aligned}\epsilon &= \sqrt{(1-\delta_p)^{-2} - 1} \\ &= \sqrt{(.9)^{-2} - 1} \\ &= .4843\end{aligned}$$

Next, from (7.4.19a)

$$\begin{aligned}\alpha &= \epsilon^{-1} + \sqrt{\epsilon^{-2} + 1} \\ &= (.4843)^{-1} + \sqrt{(.4843)^{-2} + 1} \\ &= 4.3589\end{aligned}$$

Using (7.4.19b) and (7.4.19c) with $F_0 = F_p$, the radii of the minor and major axes of the ellipse containing the poles are

$$\begin{aligned}r_1 &= \pi F_p(\alpha^{1/n} - \alpha^{-1/n}) \\ &= 10\pi((4.3589)^{1/2} - (4.3589)^{-1/2}) \\ &= 50.5427 \\ r_2 &= \pi F_p(\alpha^{1/n} + \alpha^{-1/n}) \\ &= 10\pi((4.3589)^{1/2} + (4.3589)^{-1/2}) \\ &= 80.6375\end{aligned}$$

From (7.4.20), the angles of the poles are

$$\begin{aligned}\theta_k &= \frac{(2k+1+n)\pi}{2n} \\ &= \frac{(2k+3)\pi}{4} \\ &= \{3\pi/4, 5\pi/4\}\end{aligned}$$

Using (7.4.21),the real and imaginary parts of the poles are

$\boxed{9.21}$ Consider a raised-cosine RBF network with $m = 0$, $n = 0$, $d = 2$, and $a = [0, 1]$. Using the trigonometric identities from Appendix 2, show that the constant interpolation property holds in this case. That is, show that

$$g_0(u) + g_1(u) = 1 \quad , \quad a_1 \le u \le a_2$$

Solution

When $m = 1$ and $n = 0$, the network dimension is $p = 1$. It follows from (9.9.20) that

$$g_i(u) = G\left(\frac{u - u^i}{\Delta x}\right) \quad , \quad 0 \le i \le 1$$

Since $d = 2$, from (9.9.9a) the grid point spacing is

$$\begin{aligned} \Delta x &= a_2 - a_1 \\ &= 1 \end{aligned}$$

From (9.9.8) and (9.9.11) the two grid points are

$$\begin{aligned} u_0 &= a_1 = 0 \\ u_1 &= a_2 = 1 \end{aligned}$$

Thus from (9.9.20) for $a_1 \le u \le a_2$

$$\begin{aligned} g_0(u) &= .5[1 + \cos(\pi u)] \\ g_1(u) &= .5\{1 + \cos(\pi[u - 1])\} \end{aligned}$$

Using the cosine of the difference trigonometric identity from Appendix 2 yields

$$\begin{aligned} g_0(u) + g_1(u) &= 1 + .5\cos(\pi u) + .5\cos(\pi[u - 1]) \\ &= 1 + .5\cos(\pi u) + .5[\cos(\pi u)\cos(\pi) + \sin(\pi u)\sin(\pi)] \\ &= 1 + .5\cos(\pi u) - .5\cos(\pi u) \\ &= 1 \quad , \quad a_1 \le u \le a_2 \end{aligned}$$