CONTENTS

1.5

(a) $(\frac{5}{6})^N$

(b) (Probability of being shot on the N^{th} trial) = (probability of surviving N-1 trials) $\times$ (probability of shooting oneself on the N^{th} trial) = $(\frac{5}{6})^{N-1}(\frac{1}{6})$.

(c) 6

1.6

$\overline{m} = \overline{m^3} = 0$ since these are odd moments. Using $\overline{n^r} = (p\frac{\partial}{\partial p})^r (p+q)^N$

and
$$\overline{m^2} = \overline{(2n - N)^2} = 4\,\overline{n^2} - 4\overline{n}N + N^2$$

$$\overline{m^4} = \overline{(2n - N)^4} = 16\,\overline{n^4} - 32\,N\overline{n^3} + 24\,N^2\,\overline{n^2} - 8N^3\overline{n} + N^4$$

we find $\overline{m^2} = N$ and $\overline{m^4} = 3N^2 - 2N$.

1.7

The probability of n successes out of N trials is given by the sum

$$W(n) = \sum_{i=1}^{2}\sum_{j=1}^{2}\cdots\sum_{m=1}^{2} w_i w_j \cdots w_m$$

with the restriction that the sum is taken only over terms involving w_1 n times.

Then $W(n) = \sum_{i=1}^{2} w_i \sum_{j=1}^{2} w_j \cdots \sum_{m=1}^{2} w_m$. If we sum over <u>all</u> i ... m, each sum contributes (w_1+w_2) and

$$W'(n) = (w_1 + w_2)^N$$

$$W'(n) = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} w_1^{\,n} w_2^{\,N-n} \quad \text{by the binomial theorem .}$$

Applying the restriction that w_1 must occur n times we have

$$W(n) = \frac{N!}{n!(N-n)!} w_1^{\,n} w_2^{\,N-n}$$

1.8

We consider the relative motion of the two drunks. With each simultaneous step, they have a probability of 1/4 of decreasing their separation, 1/4 of increasing it, and 1/2 of maintaining it by taking steps in the same direction. Let the number of times each case occurs be n_1, n_2, and n_3, respectively. Then the probability of a particular combination in N steps is

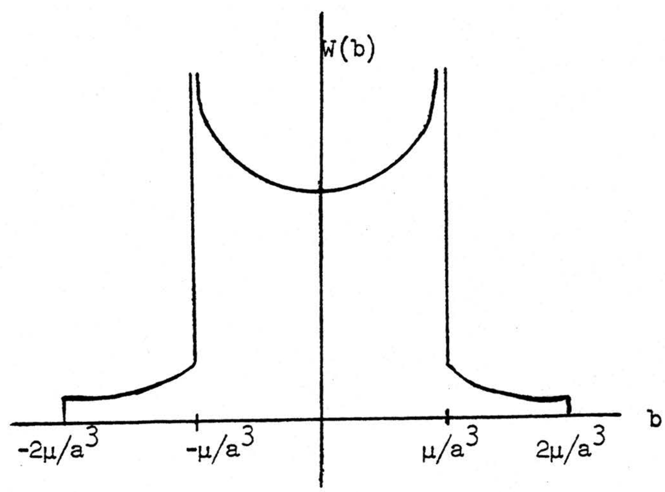
$$W(n_1\ n_2\ n_3) = \frac{N!}{n_1!n_2!n_3!}\left(\frac{1}{4}\right)^{n_1}\left(\frac{1}{4}\right)^{n_2}\left(\frac{1}{2}\right)^{n_3} \quad \text{with } n_1+n_2+n_3 = N$$

(b) If the spin is anti-parallel to the field,

$$W(b)db = \frac{a^3\sqrt{3}\ db}{6\sqrt{\mu^2-\mu\ a^3b}}$$

Then if either orientation is possible, we add the probabilities and renormalize

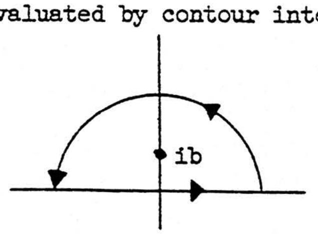
$$W(b)db = \begin{cases} \dfrac{a^3\sqrt{3}\ db}{12\sqrt{\mu^2-\mu\ a^3b}} & -\dfrac{2\mu}{a^3} < b < -\dfrac{\mu}{a^3} \\ \left(\dfrac{a^3\sqrt{3}}{12\sqrt{\mu^2-\mu\ a^3b}} + \dfrac{a^3\sqrt{3}}{12\sqrt{\mu^2+\mu a^3b}}\right) db & -\dfrac{\mu}{a^3} < b < \dfrac{\mu}{a^3} \\ \dfrac{a^3\sqrt{3}\ db}{12\sqrt{\mu^2+\mu a^3b}} & \dfrac{\mu}{a^3} < b < \dfrac{2\mu}{a^3} \end{cases}$$



1.26

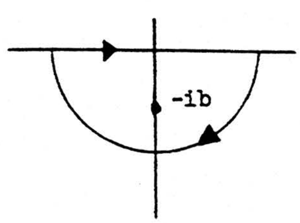
$$Q(k) = \int_{-\infty}^{\infty} ds\ e^{iks}\ W(s) = \frac{b}{\pi}\int_{-\infty}^{\infty} ds\ \frac{e^{iks}}{s^2+b^2}$$

Q may be evaluated by contour integration. For $k > 0$, the integral is evaluated on the path



$$\text{residue } (ib) = \frac{e^{-kb}}{2\pi\ i} \qquad Q(k) = e^{-kb}$$

For $k < 0$



$$-\text{residue } (-ib) = \frac{e^{kb}}{2\pi i} \qquad Q(k) = e^{kb}$$

8.8

The bar moves because the higher pressure underneath it lowers the melting point. The water is then forced up along the sides to the top of the bar and refreezes under the lower pressure. To melt a mass dm of ice, heat ℓdm must be conducted through the bar. The heat passing through it per unit time is

$$\mathcal{H} = -\kappa\ (bc)\ \frac{\Delta T}{a}$$

where ΔT is the temperature drop across the bar and κ is the thermal conductivity. Letting x be the position in the vertical direction, we have $dm = \rho_i (bc\ dx)$, and equating the rate at which heat is conducted to the rate it is absorbed in the melting process we have

$$-\kappa\ (bc)\ \frac{\Delta T}{a} = \ell\ \rho_i\ (bc\ \frac{dx}{dt}) \qquad (1)$$

ΔT is given by the Clausius-Clapeyron equation

$$\frac{dp}{dt} \approx \frac{\Delta p}{\Delta T} = \frac{\ell}{T(v_{water} - v_{ice})}$$

where v is the volume per gram or $1/\rho$. Then since $\Delta p = 2\ mg/bc$,

$$\Delta T = \frac{mg\ T}{\ell\ bc}\ (\frac{1}{\rho_w} - \frac{1}{\rho_i})$$

Substitution in (1) yields

$$\frac{dx}{dt} = \frac{2\ mg\ \kappa T}{abc\ \ell^2\ \rho_i}\ (\frac{1}{\rho_i} - \frac{1}{\rho_w})$$

8.9

From the Clausius-Clapeyron relation, $dp/dT = L/T\Delta v$ we find

$$\int \frac{dT}{T} = \int \frac{\Delta V}{L}\ dp$$

We know $L(\varphi)$ and $\Delta V(\varphi)$, and we can find $dp/d\varphi$ from the curve of vapor pressure vs. emf.

Thus
$$\int_{273.16}^{T} \frac{dT'}{T'} = \int_{o}^{\varphi} \frac{\Delta V(\varphi')}{L(\varphi')}\ (\frac{dp}{d\varphi})\ d\varphi'$$

where $273.16^{o}K$ is the temperature of the reference junction of the thermocouple.

Then
$$T = 273.16\ \exp\left[\int_{o}^{\varphi} \frac{\Delta V(\varphi')}{L(\varphi')}\ (\frac{dp}{d\varphi'})\ d\varphi'\right]$$

8.10

We divide the substance into small volume elements. The total entropy is just the sum of the entropies of each of these, or

$$S = \sum_i S_i\ (E_i, N_i)$$

where E_i and N_i denote the energy and number of particles in the i^{th} volume. If we consider

12.14

(a) $\sigma_2/\sigma_1 = (\eta_1/\eta_2)(\mu_2/\mu_1)^{1/2}$

(b) $\kappa_2/\kappa_1 = \eta_2\mu_1/\eta_1\mu_2$

(c) $D_2/D_1 = \eta_2\mu_1/\eta_1\mu_2$

(d) $\sigma = \dfrac{2}{3\sqrt{\pi}} \dfrac{1}{\eta} \sqrt{\dfrac{\mu\, kT}{N_A}}$ gives $\sigma_1 = 1.1 \times 10^{-15}$ cm^2, $\sigma_2 = 2.8 \times 10^{-15}$ cm^2.

(e) $\sigma = \pi d^2$ gives $d_1 = 1.8 \times 10^{-8}$cm, $d_2 = 3.0 \times 10^{-8}$ cm.

12.15

The gases are uniformly mixed when the mean square of the displacement is on the order of the square of the dimensions of the container. We have, after N collisions,

$$\overline{z^2} = \sum_i \overline{\zeta_i^{\,2}} + \sum_{i \neq j}\sum \overline{\zeta_i \zeta_j} = N\,\overline{\zeta^2}$$

since the second term is zero by statistical independence.

Then
$$\overline{\zeta^2} = \overline{v_z^{\,2}}\;\overline{t^2} = \frac{2}{3}\overline{v^2}\,\tau^2$$

$$\overline{z^2} = \frac{2}{3}\overline{v^2}\,\tau^2\,N = \frac{2}{3}\overline{v^2}\,\tau t$$

where $N = t/\tau$. Since $\tau = \ell/\bar{v} = 1/\sqrt{2}\, n\, \sigma\, \bar{v}$, one has on neglecting the distinction between $\bar{v}^2$ and $\overline{v^2}$,

$$\overline{z^2} = \frac{2}{3\sqrt{2}}\left(\frac{\bar{v}}{\sigma p}\, kT\right)t = \frac{\sqrt{2}}{3}\,\frac{kT}{\sigma p}\,\sqrt{\frac{8}{\pi}\frac{kT}{m}}\; t$$

Substitution of $\overline{z^2} = 10^4$ cm^2, $\sigma \approx 10^{-15}$ cm^2, and the given temperature and pressure yields $t \approx 10^4$ sec.

12.16

A molecule undergoes a momentum change of $2\mu v_z$ in a collision with the cube where $\mu = mM/m+M$, the reduced mass. Then the force is given by the momentum change per collision times the number of molecules which strike the cube per unit time, summed over all velocities.

$$\mathcal{F}_{tot} = 2\mu n \left(\frac{m\beta}{2\pi}\right)^{1/2} L^2 \int_0^{\infty} v_z^{\,2}\, e^{-\frac{\beta m}{2}(v_z - V)^2}\, dv_z$$

We may approximate the exponent since $\bar{v} \gg V$. Thus