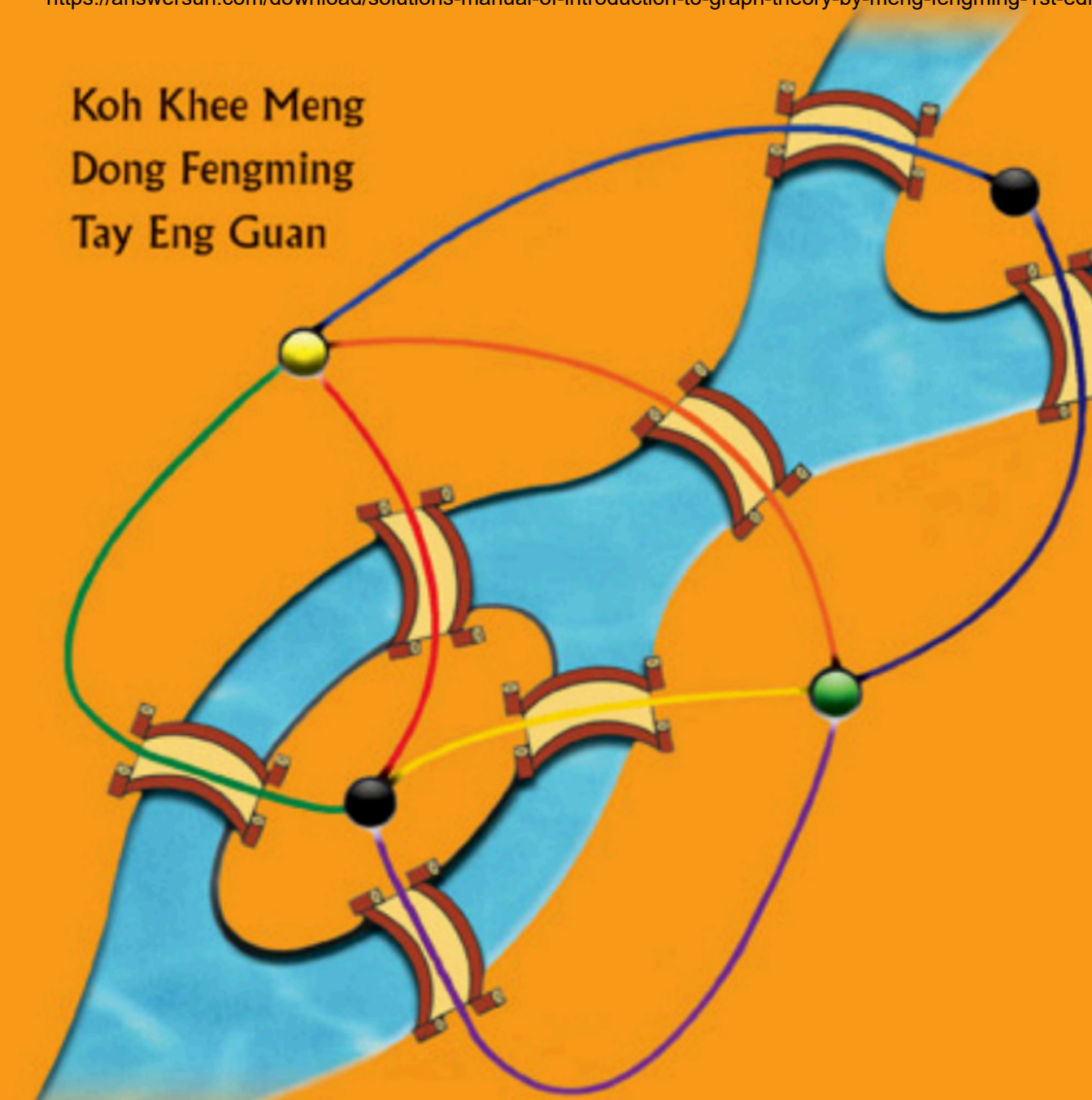


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# Introduction to Graph Theory

Solutions Manual

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## Solutions Manual

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**Problem 6.** A connected graph is said to be **unicyclic** if it contains one and only one cycle as a subgraph.

- (i) Is every cycle unicyclic?
- (ii) Construct two unicyclic graphs of order 8 which are not  $C_8$ .
- (iii) How many edges are there in each of your graphs in (ii)?

*Solution.* (i) Yes, every cycle is unicyclic.

(ii) Two such graphs are shown below:



(iii) Each graph is of size 8 (same as the order). □

**Problem 7.** Let  $G$  be a unicyclic graph.

- (i) What is the relation between  $e(G)$  and  $v(G)$ ? Justify your answer.
- (ii) Show that there exist at least three edges  $e$  in  $G$  such that  $G - e$  is a tree.

*Solution.* Let  $G$  be a unicyclic graph.

(i) Then  $e(G) = v(G)$ .

Let  $f$  be any edge contained in the only cycle in  $G$ . Observe that  $G - f$  is still connected (see Problem 16(iii) in Exercise 2.3) and it contains no cycle. Thus,  $G - f$  is a tree, and by Theorem 3.4,  $e(G - f) = v(G - f) - 1$ . It follows that  $e(G) = e(G - f) + 1 = v(G - f) = v(G)$ , as asserted.

(ii) Let  $C_k$  be the cycle in  $G$ . As shown in (i), the deletion of any edge in  $C_k$  results in a tree. The result now follows as  $k \geq 3$ . □

**Problem 8.** (+) Let  $G$  be a unicyclic graph and let  $n_1$  denote the number of end-vertices in  $G$ . Find an expression for  $n_1$  similar to that in Theorem 3.5.

*Solution.* Let  $G$  be a unicyclic graph of order  $n$  and let  $n_i$  denote the number of vertices of degree  $i$  in  $G$ ,  $i = 1, 2, \dots, k (= \Delta(G))$ . Then, by

**Problem 8.** *Let  $G$  be a graph. Determine whether each of the following statements is true.*

- (i) *If  $G$  admits a 3-colouring, then  $G$  is 3-colourable.*
- (ii) *If  $G$  is 3-colourable, then  $G$  is 5-colourable.*
- (iii) *If  $G$  is 3-colourable, then  $\chi(G) \geq 3$ .*
- (iv) *If  $G$  is 3-colourable, then  $\chi(G) \leq 3$ .*
- (v) *If  $G$  is 3-colourable, then  $G$  contains an odd cycle.*
- (vi) *If  $G$  contains an odd cycle, then  $G$  is 3-colourable.*
- (vii) *If  $G$  admits no 3-colourings, then  $\chi(G) \geq 3$ .*
- (viii) *If  $G$  admits no 3-colourings, then  $\chi(G) = 2$ .*
- (ix) *If  $G$  admits no 3-colourings, then  $\chi(G) \leq 2$ .*
- (x) *If  $\chi(G) = 3$ , then  $G$  contains a triangle.*
- (xi) *If  $\chi(G) = 3$ , then  $G$  contains an odd cycle.*
- (xii) *If  $G$  is a tree with at least two vertices, then  $\chi(G) = 2$ .*
- (xiii) *If  $\chi(G) \geq r$ , then  $G$  contains a  $K_r$  as a subgraph.*

*Solution.* (i) True.

- (ii) True.
- (iii) False.
- (iv) True
- (v) False
- (vi) False.
- (vii) True. (Indeed,  $\chi(G) \geq 4$ .)
- (viii) False.
- (ix) False.
- (x) False.
- (xi) True.
- (xii) True.
- (xiii) False.

□