Modern Particle Physics
Instructor's Manual
Version 1.03

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Preface to the instructor's manual

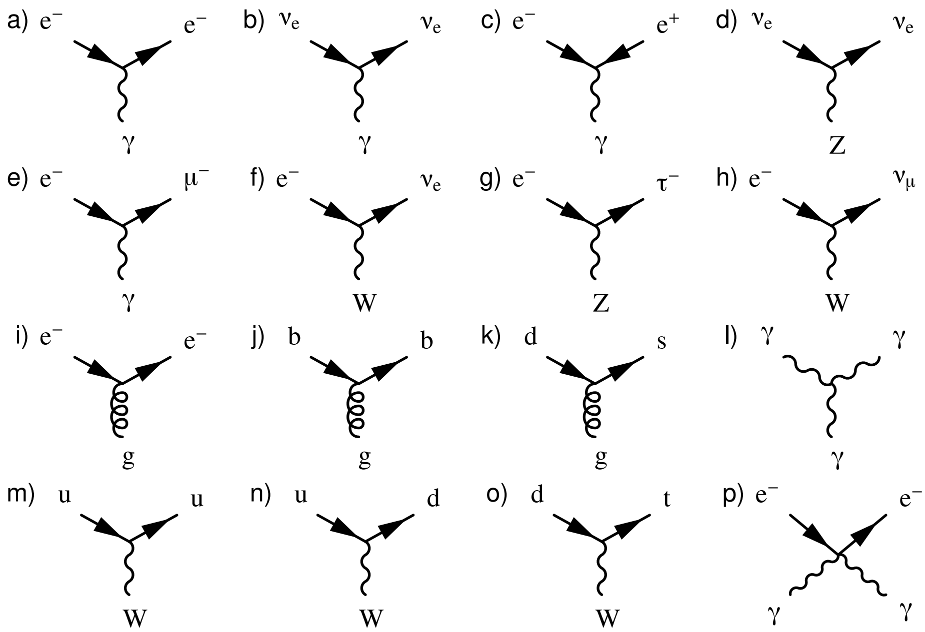
The first version of the Instructor's manual to Modern Particle Physics contains fully-worked solutions to all the problems in Chapters 1–18 of the main text. This document has not been proof-read to the extent of the main text, so I apologies in advance for any errors. Many of the problems have been used in the course that taught for a number of years, so these are battle-hardened. For new questions, introduced to address specific points in the text (particularly in the later chapters), there are a couple issues which have been noted in the solutions.

In some cases there may be more elegant approaches to the problems, the intention was to keep the solutions as straightforward as possible. Comments and suggestions are always welcome.

Mark Thomson, Cambridge, December 14th 2013

1 Introduction

1.1 Feynman diagrams are constructed out of the Standard Model vertices shown in Figure 1.4. Only the weak charged-current ($W^{\pm}$) interaction can change the flavour of the particle at the interaction vertex. Explaining your reasoning, state whether each of the sixteen diagrams below represents a valid Standard Model vertex.



The purpose of this question is to get students to understand the that (with the exception of gauge boson triple and quartic coupling) all Feynman diagrams are built out of the Standard Model three-point vertices of Figure 1.4. Of the sixteen vertices in this question, the only valid Standard Model vertices are: a), d), f), j), n) and o). The other diagrams are forbidden for the following reasons:

- b) The electron neutrino is neutral and therefore does not couple to the gauge boson of the electromagnetic interaction;
- c) This diagram violates both charge conservation and has the effect of turning a particle into an antiparticle (the arrows on the electron lines both point towards the vertex);

Because $p_\mu p_\nu \gamma^\mu \gamma^\nu$ is a symmetric tensor it can be written as $\frac{1}{2}p_\mu p_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)$ and since $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$

$$\begin{aligned} p_\mu p_\nu \gamma^\mu \gamma^\nu &= \tfrac{1}{2}p_\mu p_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) \\ &= g^{\mu\nu} p_\mu p_\nu \\ &= p_\mu p^\mu = m^2 \,. \end{aligned}$$

Thus

$$\begin{aligned} \Lambda^+\Lambda^+ &= \tfrac{1}{4m^2}(m^2 + m[\gamma^\mu p_\mu + \gamma^\nu p_\nu] + p_\mu p_\nu \gamma^\mu \gamma^\nu) \\ &= \tfrac{1}{4m^2}(2m^2 + 2m\gamma^\mu p_\mu) \\ &= \tfrac{1}{2m}(m + \gamma^\mu p_\mu) \\ &= \Lambda^+ \,. \end{aligned}$$

Similarly $\Lambda^+\Lambda^- = 0$ and $\Lambda^-\Lambda^- = \Lambda^-$.

ii) The free particle u-spinors and v-spinors respectively satisfy

$$(\gamma^\mu p_\mu - m)u = 0 \quad \text{and} \quad (\gamma^\mu p_\mu + m)u = 0\,,$$

and therefore

$$\gamma^\mu p_\mu u = +mu \quad \text{and} \quad \gamma^\mu p_\mu v = -mv\,.$$

Therefore

$$\begin{aligned} \Lambda^+ u &= \tfrac{1}{2m}(m + \gamma^\mu p_\mu)u \\ &= \tfrac{1}{2m}(m + m)u = u\,. \end{aligned}$$

The other relations follow in the same way.

6.4 Show that the helicity operator can be expressed as

$$\hat{h} = -\tfrac{1}{2}\frac{\gamma^0\gamma^5\boldsymbol{\gamma}\cdot\mathbf{p}}{\mathrm{p}}\,.$$

In the Dirac-Pauli representation, the relevant matrices are

$$\hat{S}_k = \tfrac{1}{2}\hat{\Sigma}_k = \tfrac{1}{2}\begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \text{and} \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},$$

where $k = 1, 2, 3$ and $\hat{S}_k$ are the components of the spin operator *for a Dirac spinor*.

a) Writing $u(x) = u_V(x) + S(x)$ and $d(x) = d_V(x) + S(x)$,

$$\begin{aligned}
d^2\sigma_{\mathrm{DY}}^{\mathrm{p\overline{p}}} &= \frac{4\pi\alpha^2}{81 s x_1 x_2}\{4[u_V(x_1) + S(x_1)][u_V(x_2) + S(x_2)] + 4S(x_1)S(x_2) + \\
&\qquad [d_V(x_1) + S(x_1)][d_V(x_2) + S(x_2)] + S(x_1)S(x_2)\}dx_1dx_2 \\
&= \frac{4\pi\alpha^2}{81 s x_1 x_2}\{4u_V(x_1)u_V(x_2) + 4u_V(x_1)S(x_2) + 4S(x_1)u_V(x_2) + 10S(x_1)S(x_2) + \\
&\qquad d_V(x_1)d_V(x_2) + d_V(x_1)S(x_2) + S(x_1)d_V(x_2)\}dx_1dx_2
\end{aligned}$$

If we also assume that $u_V(x) = 2d_V(x)$ then

$$\begin{aligned}
d^2\sigma_{\mathrm{DY}}^{\mathrm{p\overline{p}}} &= \frac{4\pi\alpha^2}{81 s x_1 x_2}\{17d_V(x_1)d_V(x_2) + \\
&\qquad 9d_V(x_1)S(x_2) + 9S(x_1)d_V(x_2) + 10S(x_1)S(x_2)\}dx_1dx_2 .
\end{aligned}$$

b) For pp collisions, the Drell-Yan cross section is

$$\begin{aligned}
d^2\sigma_{\mathrm{DY}}^{\mathrm{pp}} &= \frac{4\pi\alpha^2}{81 s x_1 x_2}\{4u(x_1)\overline{u}(x_2) + 4\overline{u}(x_1)u(x_2) + \\
&\qquad d(x_1)\overline{d}(x_2) + \overline{d}(x_1)d(x_2)\}dx_1dx_2
\end{aligned}$$

This can be expressed in terms of sea and valance quark pdfs:

$$\begin{aligned}
d^2\sigma_{\mathrm{DY}}^{\mathrm{pp}} &= \frac{4\pi\alpha^2}{81 s x_1 x_2}\{4u_V(x_1)S(x_2) + 4S(x_1)u_V(x_2) + \\
&\qquad d_V(x_1)S(x_2) + S(x_1)d_V(x_2) + 10S(x_1)S(x_2)\}dx_1dx_2
\end{aligned}$$

If we again assume that $u_V(x) = 2d_V(x)$ then

$$d^2\sigma_{\mathrm{DY}}^{\mathrm{pp}} = \frac{4\pi\alpha^2}{81 s x_1 x_2}\{9d_V(x_1)S(x_2) + 9S(x_1)d_V(x_2) + 10S(x_1)S(x_2)\}dx_1dx_2$$

c) Since $\hat{s} = x_1x_2s$, lines of constant $\hat{s}$ define hyperbolae in the $\{x_1, x_2\}$ plane. For $\hat{s} \ll s$ both x_1 and x_2 will usually be small, and in this region the Drell-Yen cross section will be dominated the sea quarks and, from the above results, $d^2\sigma_{\mathrm{DY}}^{\mathrm{p\overline{p}}} \sim \sigma_{\mathrm{DY}}^{\mathrm{pp}}$. Consequently the cross section for the Drell-Yan production of low-mass $\mu^+\mu^-$-pairs will be approximately the same for pp and $\mathrm{p\overline{p}}$ collisions. In contrasts for $\hat{s} > s/4$, both x_1 and x_2 will be greater than 0.5 and the valance quark contributions will dominate over the sea. In this case $d^2\sigma_{\mathrm{DY}}^{\mathrm{p\overline{p}}} \gg \sigma_{\mathrm{DY}}^{\mathrm{pp}}$, and the cross section for the production of high-mass $\mu^+\mu^-$-pairs will be much greater for $\mathrm{p\overline{p}}$ collisions.

10.9 Drell-Yan production of $\mu^+\mu^-$-pairs with an invariant mass Q^2 has been studied in $\pi^\pm$ interactions with Carbon (which has equal numbers of protons and neutrons). Explain why the ratio

$$\frac{\sigma(\pi^+\mathrm{C} \to \mu^+\mu^- X)}{\sigma(\pi^-\mathrm{C} \to \mu^+\mu^- X)}$$

and the Euler-Lagrange equation gives:

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)-\frac{\partial\mathcal{L}}{\partial\phi_i}=0$$
$$\partial_\mu i\overline{\psi}\gamma^\mu+m\overline{\psi}=0$$
$$i(\partial_\mu\overline{\psi})\gamma^\mu+m\overline{\psi}=0\,.$$

17.3 Verify that the Lagrangian for the free electromagnetic field,

$$\mathcal{L}=-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\,,$$

is invariant under the gauge transformation $A_\mu\to A'_\mu=A_\mu-\partial_\mu\chi$.

The field strength tensor

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\,,$$

transforms to

$$\begin{aligned}F^{\mu\nu\prime}&=\partial^\mu(A^\nu-\partial^\nu\chi)-\partial^\nu(A^\mu-\partial^\mu\chi)\\&=\partial^\mu A^\nu-\partial^\mu\partial^\nu\chi-\partial^\nu A^\mu+\partial^\nu\partial^\mu\chi\\&=\partial^\mu A^\nu-\partial^\nu A^\mu\\&=F^{\mu\nu}\,,\end{aligned}$$

and consequently $F^{\mu\nu}F_{\mu\nu}$ is invariant under the gauge transformation.

17.4 The Lagrangian for the electromagnetic field in the presence of a current j^μ is

$$\mathcal{L}=-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}-j^\mu A_\mu\,.$$

By writing this as

$$\begin{aligned}\mathcal{L}&=-\tfrac{1}{4}(\partial^\mu A^\nu-\partial^\nu A^\mu)(\partial_\mu A_\nu-\partial_\nu A_\mu)-j^\mu A_\mu\\&=-\tfrac{1}{2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)+\tfrac{1}{2}(\partial^\nu A^\mu)(\partial_\mu A_\nu)-j^\mu A_\mu\,,\end{aligned}$$

show that the Euler-Lagrange equation gives the covariant form of Maxwell's equations,

$$\partial_\mu F^{\mu\nu}=j^\nu\,.$$

The Lagrangian density

$$\begin{aligned}\mathcal{L}&=-\tfrac{1}{4}(\partial^\mu A^\nu-\partial^\nu A^\mu)(\partial_\mu A_\nu-\partial_\nu A_\mu)-j^\mu A_\mu\\&=-\tfrac{1}{4}(\partial^\mu A^\nu)(\partial_\mu A_\nu)-\tfrac{1}{4}(\partial^\nu A^\mu)(\partial_\nu A_\mu)+\tfrac{1}{4}(\partial^\mu A^\nu\partial_\nu A_\mu)+\tfrac{1}{4}(\partial^\nu A^\mu)(\partial_\mu A_\nu)-j^\mu A_\mu\\&=-\tfrac{1}{2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)+\tfrac{1}{2}(\partial^\nu A^\mu)(\partial_\mu A_\nu)-j^\mu A_\mu\\&=-\tfrac{1}{2}(\partial^\nu A^\mu)(\partial_\nu A_\mu)+\tfrac{1}{2}(\partial^\nu A^\mu)(\partial_\mu A_\nu)-j^\mu A_\mu\,.\end{aligned}$$