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Solutions Manual to Accompany

ORDINARY DIFFERENTIAL EQUATIONS

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WILEY

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$$y_2(x) = x^2 \int \frac{e^{-\int (-3/x)\,dx}}{x^4}\,dx = x^2 \int \frac{e^{3\ln x}}{x^4}\,dx = x^2 \int \frac{x^3}{x^4}\,dx = \boldsymbol{x^2 \ln x}.$$

Now re-work the problem using the method instead. Since $y_1(x) = x^2$, seek a second solution in the form $y(x) = C(x)x^2$. Substituting that in the DE gives

$$x^2(C'' + 4xC' + 2C) - 3x(C'x^2 + 2xC) + 4(Cx^2) = 0,$$

which simplifies to $xC'' + C' = 0$. We could set $C' = v$ to reduce the order of the latter, but it is easier to notice that it can be written as $(xC')' = 0$. which gives $xC' = A$ and $C(x) = A\ln x + B$. Thus, we have found $y(x) = (B + A\ln x)x^2$. Take $B = 0$ because that term gives the solution $y_1(x) = x^2$ that was given. Thus, we have found $y_2(x) = \boldsymbol{x^2 \ln x}$ again.

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$$x' = 2x + 2z,$$
$$y' = x + y + 2z,$$
$$z' = x + 3z.$$

SOLUTION. $\boldsymbol{A} = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ gives $\lambda_1 = 1$, $\boldsymbol{e}_1 = \alpha \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$; $\lambda_2 = 4$, $\boldsymbol{e}_2 = \gamma \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$;
so a general solution is

$$\boldsymbol{x}(t) = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{t} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{4t}.$$

Example 2. (Defective eigenvalues) Obtain a general solution of the system

$$x' = 2x + 4y,$$
$$y' = -x + 6y.$$

We find that the matrix $\boldsymbol{A} = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix}$ gives $\lambda = 4, 4$ with only the one-dimensional eigenspace $\boldsymbol{e} = \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Thus, seeking solutions in the form $\boldsymbol{x}(t) = q\, e^{rt}$ comes up short, giving only

$$\boldsymbol{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{4t} + c_2 \times ?. \tag{A}$$

Since the defect is 1 (that is, the multiplicity 2 of the eigenvalue minus the dimension 1 of the eigenspace), seek $\boldsymbol{x}(t)$ in the modified form

$$\boldsymbol{x}(t) = (\boldsymbol{q} + \boldsymbol{p}\, t) e^{4t}. \tag{B}$$

Putting (B) into $\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}$ gives

$$4\boldsymbol{q}\, e^{4t} + \boldsymbol{p}\, e^{4t} + 4\boldsymbol{p}\, t\, e^{4t} = \boldsymbol{A}\boldsymbol{q}\, e^{4t} + \boldsymbol{A}\boldsymbol{p}\, t\, e^{4t}. \tag{C}$$

Cancel the exponentials and then use the linear independence of 1 and t to match their coefficients on the two sides of equation (C):

$$t: \quad \boldsymbol{A}\boldsymbol{p} = 4\boldsymbol{p}, \tag{D}$$
$$1: \quad \boldsymbol{A}\boldsymbol{q} = 4\boldsymbol{q} + \boldsymbol{p}. \tag{E}$$

(D) is the eigenvalue problem for the matrix $\boldsymbol{A}$, with eigenvalue 4, that we've already solved, above, so $\boldsymbol{p}$ is the already-found eigenvector, $\boldsymbol{p} = \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Next, put that $\boldsymbol{p}$ into (E), which becomes

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$$y(x) = x^r \sum_0^\infty a_n x^n = \sum_0^\infty a_n x^n \tag{2}$$

will lead to one solution, $y_1(x)$; then, text equation (18b) will give a second LI solution $y_2(x)$. First find $y_1(x)$: Putting (2) into (1) gives

$$\sum_2^\infty a_n n\,(n-1)x^{n-1} + \sum_1^\infty a_n n\, x^{n-1} - \sum_0^\infty a_n x^{n+1} = 0.$$

To get the exponents the same, set $n-1=k$ in the first and second sums, and $n+1=k$ in the third, so

$$\sum_1^\infty a_{k+1}(k+1)k\,x^k + \sum_0^\infty a_{k+1}(k+1)x^k - \sum_1^\infty a_{k-1}x^k = 0.$$

Now to get the same summation limits, we can change the lower limit in the first sum to 0 without harm because the k factor is zero at $k=0$ anyhow; and we can change the lower limit in the third sum to 0 with the understanding that $a_{-1} \equiv 0$. Thus,

$$\sum_0^\infty \left[a_{k+1}(k+1)^2 - a_{k-1}\right]x^k = 0.$$

The latter gives the recursion formula as $a_{k+1}(k+1)^2 - a_{k-1} = 0$, or

$$a_{k+1} = \frac{a_{k-1}}{(k+1)^2} \quad \text{for } k = 0, 1, 2, \ldots, \tag{3}$$

with $a_{-1} \equiv 0$. Putting $k = 0, 1, 2, \ldots$ in (3), gives $a_1 = 0$, $a_2 = \frac{1}{4}a_0$, $a_3 = 0$, $a_4 = \frac{1}{16}a_2 = \frac{1}{64}a_0$, and so on, with a_0 remaining arbitrary. Thus,

$$\begin{aligned} y_1(x) &= \sum_0^\infty a_n x^n = a_0 + \frac{1}{4}a_0 x^2 + \frac{1}{64}a_0 x^4 + \cdots \\ &= 1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + \cdots \end{aligned} \tag{4}$$

in which we've taken $a_0 = 1$.

Next, according to (18b) in the text, seek a second solution in the form

$$y_2(x) = y_1(x)\ln x + \sum_1^\infty c_n x^n. \tag{5}$$

[Note that the lower summation limit in (5) is 1, not 0.] Differentiate (5) twice, to put it into (1):

$$y_2'(x) = y_1'(x)\ln x + \frac{1}{x}y_1(x) + \sum_1^\infty n c_n x^{n-1},$$

$$y_2''(x) = y_1''(x)\ln x + \frac{2}{x}y_1'(x) - \frac{1}{x^2}y_1(x) + \sum_2^\infty n(n-1)c_n x^{n-2},$$

and putting these into (1) gives

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continuous on I. Thus, we must be able to test a solution set $\{\boldsymbol{x}_1(t), \ldots, \boldsymbol{x}_n(t)\}$ to see if it is LI. It is LI on I if and only if

$$\det\,[\boldsymbol{x}_1(t), \ldots, \boldsymbol{x}_n(t)] \neq 0 \tag{14}$$

for all t in I. We can evaluate the determinant in (14), above, using the *Maple* **Determinant** command within the LinearAlgebra package. To illustrate, let us verify that the two vector solutions given in Example 2 of Section 4.5 are LI on the interval $t > 0$. To test them, make those vectors the columns of a 2×2 matrix and evaluate its determinant (first by hand) as

$$\det\,[\boldsymbol{x}_1, \boldsymbol{x}_2] = \begin{vmatrix} e^t & e^{2t} \\ -3e^t & -2e^{2t} \end{vmatrix} = e^t e^{2t} \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} = e^{3t}, \tag{15}$$

and since the latter is nonzero on the interval I, the two vector solutions are indeed LI on I. In this case, hand calculation sufficed because the determinant was only 2×2, but for larger determinants it is generally more convenient to use *Maple.* To illustrate, the *Maple* commands for the evaluation of the determinant given above are these:

```
> with(LinearAlgebra):
> B:=Matrix([[exp(t),exp(2*t)],[-3*exp(t),-2*exp(2*t)]])
> Determinant(B)
```

which gives the same result, e^{3t}. Remember that you can suppress printing, following a command, by using a colon at the end of the command, but it is good to not do that at first, here for the B matrix, until it prints, so we have a chance to check it for typographical errors. If it looks okay, *then* you can put the colon at the end and rerun that command, to suppress the printing so your work is more compact, if you wish. Note, as above, the format: Matrix([[first row], ... ,[last row]]).

Section 4.6. The **Eigenvectors** command, also within LinearAlgebra, gives both the eigenvalues and eigenvectors of a matrix. For instance, let

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}. \tag{16}$$

The commands

```
> with(LinearAlgebra):
> A:=Matrix([[3,4],[2,1]])
> Eigenvectors(A)
```

give as output a column vector followed by a matrix: $\begin{bmatrix} -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. The elements of the column vector are the eigenvalues, and the columns of the matrix are corresponding eigenvectors. In this

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