Solutions Manual to accompany

Elementary Linear Programming with Applications

SECOND EDITION

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10. Denote the entries of the identity matrix by d_{ij}, so that

$$d_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Then for $\mathbf{C} = \mathbf{AI_n}$, $c_{ij} = \sum_{k=1}^{p} a_{ik}d_{kj} = a_{ij}d_{jj}$ (all other d_{kj} are zero), $= a_{ij}$ and thus $\mathbf{C} = \mathbf{A}$. A similar argument shows that $\mathbf{I_m A} = \mathbf{A}$.

11. Let $\mathbf{B} = [b_{ij}] = (-1)\,\mathbf{A}$. Then $b_{ij} = -a_{ij}$.

12. $\mathbf{AB} = \begin{bmatrix} 16 & 17 & 12 & 26 & 20 \\ 6 & 19 & -4 & 18 & 17 \\ 26 & 33 & 18 & 52 & 46 \\ 5 & 16 & -5 & 15 & 9 \\ 3 & 15 & -9 & 17 & 11 \\ 18 & 29 & 12 & 38 & 33 \end{bmatrix}$

13. Let $\mathbf{A} = [a_{ij}]$ be $m \times p$ and $\mathbf{B} = [b_{ij}]$ be $p \times n$.

(a) Let the ith row of $\mathbf{A}$ consist entirely of zeros, so $a_{ik} = 0$ for $k = 1$, 2, ..., p. Then the (i, j) entry in $\mathbf{AB}$ is

$$\sum_{k=1}^{p} a_{ik}b_{kj} = 0 \text{ for } j = 1, 2, \ldots, n.$$

(b) Let the jth column of $\mathbf{B}$ consist entirely of zeros, so $b_{kj} = 0$ for $k = 1, 2, \ldots, p$. Then again the (i, j) entry in $\mathbf{AB}$ is 0 for $i = 1, 2, \ldots, m$.

14. The jth column of $\mathbf{AB}$ is

$$\begin{bmatrix} \sum_k a_{1k}b_{kj} \\ \sum_k a_{2k}b_{kj} \\ \vdots \\ \sum_k a_{mk}b_{kj} \end{bmatrix}.$$

of $\mathbf{I}_n$ to the jth row of $\mathbf{I}_n$. Let $\mathbf{E}_3^*$ be obtained from $\mathbf{I}_n$ by adding $-c$ times the ith row of $\mathbf{I}_n$ to the jth row of $\mathbf{I}_n$. Then $\mathbf{E}_3\mathbf{E}_3^* = \mathbf{I}_n$.

15. Suppose that $\mathbf{A}$ is nonsingular. Then multiplying both sides of $\mathbf{Ax} = \mathbf{b}$ on the left by $\mathbf{A}^{-1}$, we obtain

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}.$$

Conversely, suppose that $\mathbf{Ax} = \mathbf{b}$ has a solution for every $n \times 1$ matrix $\mathbf{b}$. Let $\mathbf{e}_i$ be the $n \times 1$ matrix with a 1 in the ith row and 0's elsewhere. Then the linear system $\mathbf{Ax} = \mathbf{e}_i$ has a solution $\mathbf{x}_i$. Let $\mathbf{B}$ be the $n \times n$ matrix whose jth column is $\mathbf{e}_j$. The equations $\mathbf{Ax}_1 = \mathbf{e}_1, \mathbf{Ax}_2 = \mathbf{e}_2, \ldots,$ $\mathbf{Ax}_n = \mathbf{e}_n$ can be written in matrix form as

$$\mathbf{AB} = \mathbf{I_n}.$$

Hence, $\mathbf{B}$ is the inverse of $\mathbf{A}$ and thus $\mathbf{A}$ is nonsingular.

16. The matrices $\mathbf{A}$ and $\mathbf{B}$ are row equivalent if and only if

$$\mathbf{B} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A}.$$

Let $\mathbf{P} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1$.

17. If $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$, then $\mathbf{I}_n = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A}$, where $\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_k$ are elementary matrices. Therefore, it follows that $\mathbf{A} = \mathbf{E}_1^{-1}\mathbf{E}_2^{-1}\cdots\mathbf{E}_k^{-1}$. Now the inverse of an elementary matrix is an elementary matrix. By Theorem 0.6, $\mathbf{A}$ is nonsingular.

Conversely, if $\mathbf{A}$ is nonsingular, then $\mathbf{A}$ is a product of elementary matrices, $\mathbf{A} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1$ Now

$$\mathbf{A} = \mathbf{AI}_n = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{I}_n,$$

which implies that $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$.

Section 0.4, page 32

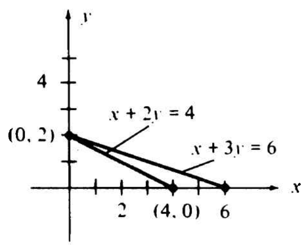
1. $\mathbf{0} + \mathbf{0} = \mathbf{0}$ and $r\mathbf{0} = \mathbf{0}$, where r is any real number.

37. Let $\mathbf{x}_1$ and $\mathbf{x}_2 \in R^n$. Then

$$\begin{aligned} f(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) &= \mathbf{c}^T\mathbf{x} \\ &= \mathbf{c}^T(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \\ &= \lambda\mathbf{c}^T\mathbf{x}_1 + (1-\lambda)\mathbf{c}^T\mathbf{x}_2 \\ &= \lambda f(\mathbf{x}_1) + (1-\lambda)f(\mathbf{x}_2) \end{aligned}$$

Section 1.4, page 90

2. (a)



$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$; $z = -6$

	x_1	x_2	x_3	x_4	x_5	
x_3	0	4	1	-1	0	4
x_1	1	-2	0	1	0	4
$\leftarrow$ x_5	3	8	0	-3	1	0
	0	-14	0	5	0	20

	x_1	x_2	x_3	x_4	x_5	
$\leftarrow$ x_3	0	0	1	$\frac{1}{2}$	$-1/2$	4
x_1	1	0	0	$1/4$	$1/4$	4
x_2	0	1	0	$-3/8$	$1/8$	0
	0	0	0	$-1/4$	$7/4$	20

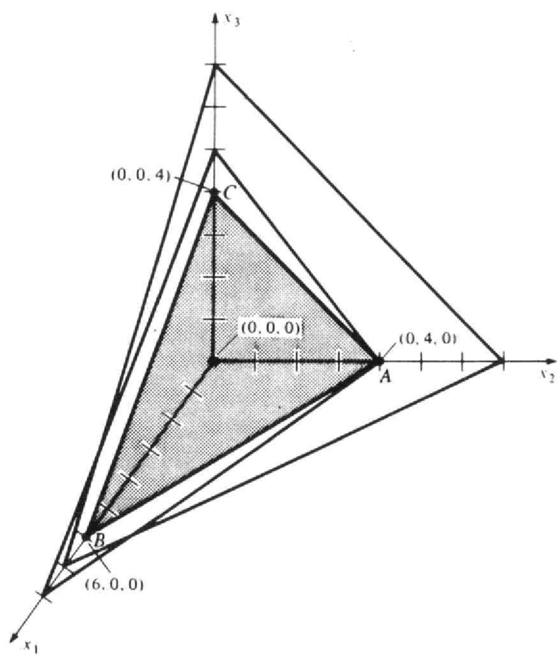
	x_1	x_2	x_3	x_4	x_5	
x_4	0	0	2	1	-1	8
x_1	1	0	$-1/2$	0	$1/2$	2
x_2	0	1	$3/4$	0	$-1/4$	3
	0	0	$1/2$	0	$3/2$	22

Note that in the first tableau, x_5 could also have been chosen as the departing variable.

4. $[0 \quad 4 \quad 0]^T$; $\quad z = 32$

The simplex algorithm examines the following extreme points: O, A, A

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	1	1	1	1	0	0	7
x_5	2	3	3	0	1	0	12
$\leftarrow$ x_6	3	6	5	0	0	1	24
	-5	-8	-1	0	0	0	0



subject to

$$\begin{array}{rcrcl} 30w_1 & - & w_2 & \leq & 15,000 \\ 50w_1 & - & 2w_2 & \leq & 20,000 \end{array}$$

$w_1 \geq 0,\ w_2 \geq 0$

10. Minimize $z' = 420w_1 + 600w_2$
subject to

$$\begin{array}{rcrcl} 2w_1 & + & 4w_2 & \geq & 0.5 \\ 2w_1 & + & 6w_2 & \geq & 0.8 \\ 3w_1 & + & 10w_2 & \geq & 1.2 \end{array}$$

$w_1 \geq 0,\ w_2 \geq 0$

where w_1 and w_2 represent the marginal values of the sewing and gluing processes, respectively.

11. Let $\mathbf{x}' = \mathbf{u} - \mathbf{v},\ \mathbf{u} \geq \mathbf{0},\ \mathbf{v} \geq \mathbf{0}$. The given problem can be written as

$$\text{Maximize } z = [\mathbf{c}^T \ \ \mathbf{d}^T \ \ -\mathbf{d}^T] \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix}$$

subject to

$$[\mathbf{A} \ \ \mathbf{B} \ \ -\mathbf{B}] \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix} \leq \mathbf{b}$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix} \geq \mathbf{0}$$

By definition the dual is

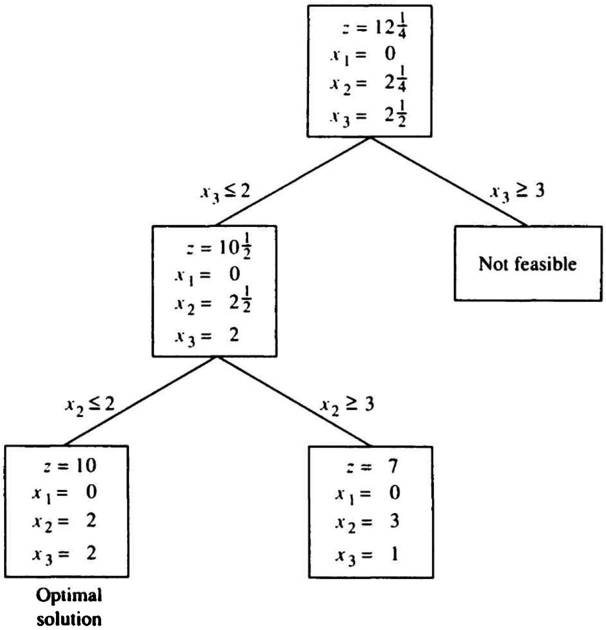
Minimize $z' = \mathbf{b}^T\mathbf{w}$
subject to

$$\begin{bmatrix} \mathbf{A}^T \\ \mathbf{B}^T \\ -\mathbf{B}^T \end{bmatrix} \mathbf{w} \geq \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \\ -\mathbf{d} \end{bmatrix}$$

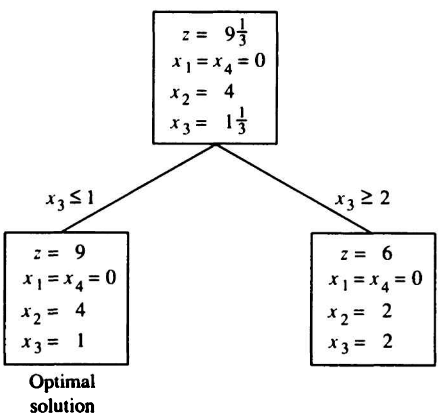
$\mathbf{w} \geq \mathbf{0}$

or multiplying out we have

4.


$z = 12\frac{1}{4}$
$x_1 = 0$
$x_2 = 2\frac{1}{4}$
$x_3 = 2\frac{1}{2}$
$x_3 \le 2$
$x_3 \ge 3$
$z = 10\frac{1}{2}$
$x_1 = 0$
$x_2 = 2\frac{1}{2}$
$x_3 = 2$
Not feasible
$x_2 \le 2$
$x_2 \ge 3$
$z = 10$
$x_1 = 0$
$x_2 = 2$
$x_3 = 2$
Optimal solution
$z = 7$
$x_1 = 0$
$x_2 = 3$
$x_3 = 1$

6.


$z = 9\frac{1}{3}$
$x_1 = x_4 = 0$
$x_2 = 4$
$x_3 = 1\frac{1}{3}$
$x_3 \le 1$
$x_3 \ge 2$
$z = 9$
$x_1 = x_4 = 0$
$x_2 = 4$
$x_3 = 1$
Optimal solution
$z = 6$
$x_1 = x_4 = 0$
$x_2 = 2$
$x_3 = 2$

Section 5.6, page 385

2. (a,b)

Node	Early event time	Late event time
1	0	0
2	5	5
3	12	15
4	8	8
5	17	17
6	18	19
7	19	19
8	25	25

(c) $1 \to 4 \to 5 \to 7 \to 8$

4. (a,b)

Node	Early event time	Late event time
1	0	0
2	3	6
3	5	6
4	4	4
5	7	12
6	5	8
7	5	5
8	10	13
9	9	10
10	9	14
11	10	10
12	11	19
13	17	17
14	12	20
15	19	22
16	23	23
17	27	27

(c) $1 \to 4 \to 7 \to 11 \to 13 \to 16 \to 17$