

Instructor's Solution Manual
Introduction to Electrodynamics
Fourth Edition

David J. Griffiths

2014

Contents

1	Vector Analysis	4
2	Electrostatics	26
3	Potential	53
4	Electric Fields in Matter	92
5	Magnetostatics	110
6	Magnetic Fields in Matter	133
7	Electrodynamics	145
8	Conservation Laws	168
9	Electromagnetic Waves	185
10	Potentials and Fields	210
11	Radiation	231
12	Electrodynamics and Relativity	262

Preface

Although I wrote these solutions, much of the typesetting was done by Jonah Gollub, Christopher Lee, and James Terwilliger (any mistakes are, of course, entirely their fault). Chris also did many of the figures, and I would like to thank him particularly for all his help. If you find errors, please let me know (griffith@reed.edu).

David Griffiths

Chapter 1

Vector Analysis

Problem 1.1

- (a) From the diagram, $|\mathbf{B} + \mathbf{C}| \cos \theta_3 = |\mathbf{B}| \cos \theta_1 + |\mathbf{C}| \cos \theta_2$. Multiply by $|\mathbf{A}|$.

$$|\mathbf{A}| |\mathbf{B} + \mathbf{C}| \cos \theta_3 = |\mathbf{A}| |\mathbf{B}| \cos \theta_1 + |\mathbf{A}| |\mathbf{C}| \cos \theta_2.$$

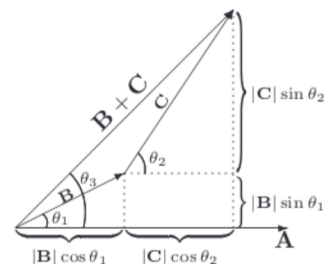
So: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$. (Dot product is distributive)

Similarly: $|\mathbf{B} + \mathbf{C}| \sin \theta_3 = |\mathbf{B}| \sin \theta_1 + |\mathbf{C}| \sin \theta_2$. Multiply by $|\mathbf{A}| \hat{\mathbf{n}}$.

$$|\mathbf{A}| |\mathbf{B} + \mathbf{C}| \sin \theta_3 \hat{\mathbf{n}} = |\mathbf{A}| |\mathbf{B}| \sin \theta_1 \hat{\mathbf{n}} + |\mathbf{A}| |\mathbf{C}| \sin \theta_2 \hat{\mathbf{n}}.$$

If $\hat{\mathbf{n}}$ is the unit vector pointing out of the page, it follows that

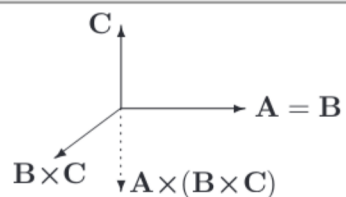
$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C}). \quad (\text{Cross product is distributive})$$



- (b) For the general case, see G. E. Hay's *Vector and Tensor Analysis*, Chapter 1, Section 7 (dot product) and Section 8 (cross product)

Problem 1.2

The triple cross-product is *not* in general associative. For example, suppose $\mathbf{A} = \mathbf{B}$ and \mathbf{C} is perpendicular to \mathbf{A} , as in the diagram. Then $(\mathbf{B} \times \mathbf{C})$ points out-of-the-page, and $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ points *down*, and has magnitude ABC . But $(\mathbf{A} \times \mathbf{B}) = \mathbf{0}$, so $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \mathbf{0} \neq \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$.

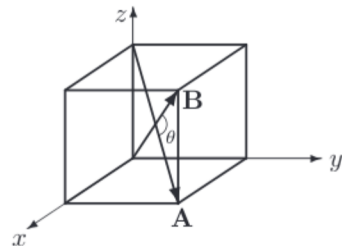


Problem 1.3

$$\mathbf{A} = +1\hat{\mathbf{x}} + 1\hat{\mathbf{y}} - 1\hat{\mathbf{z}}; \quad A = \sqrt{3}; \quad \mathbf{B} = 1\hat{\mathbf{x}} + 1\hat{\mathbf{y}} + 1\hat{\mathbf{z}}; \quad B = \sqrt{3}.$$

$$\mathbf{A} \cdot \mathbf{B} = +1 + 1 - 1 = 1 = AB \cos \theta = \sqrt{3}\sqrt{3} \cos \theta \Rightarrow \cos \theta = \frac{1}{3}.$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5288^\circ$$



Problem 1.4

The cross-product of any two vectors in the plane will give a vector perpendicular to the plane. For example, we might pick the base (\mathbf{A}) and the left side (\mathbf{B}):

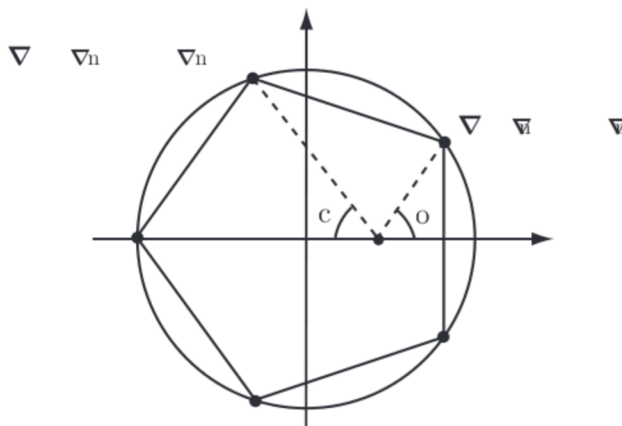
$$\mathbf{A} = -1\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 0\hat{\mathbf{z}}; \quad \mathbf{B} = -1\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 3\hat{\mathbf{z}}.$$

The even powers cancel, leaving

$$8w - 16w^3 + 2w^5 + 4w^7 = 0, \quad \text{or} \quad 4 - 8v + v^2 + 2v^3 = 0,$$

where $v \equiv w^2$. According to Mathematica, this cubic equation has one negative root, one root that is spurious (the point lies outside the square), and $v = 0.598279$, which yields

$$r = \sqrt{\frac{v}{2}} a = \boxed{0.546936 a}.$$



For the pentagon:

$$E_x = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(a+r)^2} + 2\frac{\cos\theta}{b^2} - 2\frac{\cos\phi}{c^2} \right) = 0,$$

where

$$\cos\theta = \frac{a\cos(2\pi/5) + r}{b}, \quad \cos\phi = \frac{a\cos(\pi/5) - r}{c};$$

$$b^2 = [a\cos(2\pi/5) + r]^2 + [a\sin(2\pi/5)]^2 = a^2 + r^2 + 2ar\cos(2\pi/5),$$

$$c^2 = [a\cos(\pi/5) - r]^2 + [a\sin(\pi/5)]^2 = a^2 + r^2 - 2ar\cos(\pi/5).$$

$$\frac{1}{(a+r)^2} + 2\frac{r + a\cos(2\pi/5)}{[a^2 + r^2 + 2ar\cos(2\pi/5)]^{3/2}} + 2\frac{r - a\cos(\pi/5)}{[a^2 + r^2 - 2ar\cos(\pi/5)]^{3/2}} = 0.$$

Mathematica gives the solution $r = 0.688917 a$.

For an n -sided regular polygon there are evidently n such points, lying on the radial spokes that bisect the sides; their distance from the center appears to grow monotonically with n : $r(3) = 0.285$, $r(4) = 0.547$, $r(5) = 0.689$, \dots . As $n \rightarrow \infty$ they fill out a circle that (in the limit) coincides with the ring of charge itself.

Problem 2.59 The theorem is *false*. For example, suppose the conductor is a neutral sphere and the external field is due to a nearby positive point charge q . A negative charge will be induced on the near side of the sphere (and a positive charge on the far side), so the force will be *attractive* (toward q). If we now reverse the sign of q , the induced charges will also reverse, but the force will still be attractive.

If the external field is *uniform*, then the net force on the induced charges is zero, and the total force on the conductor is QE_e , which *does* switch signs if E_e is reversed. So the “theorem” is valid in this very special case.

Combining these results: $a = 5\epsilon_0 (R^4 b_5 + R^4 b_5) = 10\epsilon_0 R^4 b_5$; $b_5 = \frac{a}{10\epsilon_0 R^4}$; $d_5 = \frac{aR^6}{10\epsilon_0}$. Therefore

$$V(s, \phi) = \frac{a \sin 5\phi}{10\epsilon_0} \begin{cases} s^5/R^4, & \text{for } s < R, \\ R^6/s^5, & \text{for } s > R. \end{cases}$$

Problem 3.27 Since \mathbf{r} is on the z axis, the angle α is just the polar angle θ (I'll drop the primes, for simplicity).

Monopole term:

$$\int \rho d\tau = kR \int \left[\frac{1}{r^2} (R - 2r) \sin \theta \right] r^2 \sin \theta dr d\theta d\phi.$$

But the r integral is

$$\int_0^R (R - 2r) dr = (Rr - r^2)|_0^R = R^2 - R^2 = 0.$$

So the monopole term is zero.

Dipole term:

$$\int r \cos \theta \rho d\tau = kR \int (r \cos \theta) \left[\frac{1}{r^2} (R - 2r) \sin \theta \right] r^2 \sin \theta dr d\theta d\phi.$$

But the θ integral is

$$\int_0^\pi \sin^2 \theta \cos \theta d\theta = \left. \frac{\sin^3 \theta}{3} \right|_0^\pi = \frac{1}{3}(0 - 0) = 0.$$

So the dipole contribution is likewise zero.

Quadrupole term:

$$\int r^2 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \rho d\tau = \frac{1}{2} kR \int r^2 (3 \cos^2 \theta - 1) \left[\frac{1}{r^2} (R - 2r) \sin \theta \right] r^2 \sin \theta dr d\theta d\phi.$$

r integral:

$$\int_0^R r^2 (R - 2r) dr = \left(\frac{r^3}{3} R - \frac{r^4}{2} \right) \Big|_0^R = \frac{R^4}{3} - \frac{R^4}{2} = -\frac{R^4}{6}.$$

θ integral:

$$\begin{aligned} \int_0^\pi \underbrace{(3 \cos^2 \theta - 1)}_{3(1 - \sin^2 \theta) - 1 = 2 - 3 \sin^2 \theta} \sin^2 \theta d\theta &= 2 \int_0^\pi \sin^2 \theta d\theta - 3 \int_0^\pi \sin^4 \theta d\theta \\ &= 2 \left(\frac{\pi}{2} \right) - 3 \left(\frac{3\pi}{8} \right) = \pi \left(1 - \frac{9}{8} \right) = -\frac{\pi}{8}. \end{aligned}$$

ϕ integral:

$$\int_0^{2\pi} d\phi = 2\pi.$$

The whole integral is:

$$\frac{1}{2} kR \left(-\frac{R^4}{6} \right) \left(-\frac{\pi}{8} \right) (2\pi) = \frac{k\pi^2 R^5}{48}.$$