Students' Solutions Manual

for

Applied Linear Algebra

by

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Second Edition

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To the Student

These solutions are a resource for students studying the second edition of our text *Applied Linear Algebra*, published by Springer in 2018. An expanded solutions manual is available for registered instructors of courses adopting it as the textbook.

Using the Manual

The material taught in this book requires an active engagement with the exercises, and we urge you not to read the solutions in advance. Rather, you should use the ones in this manual as a means of verifying that your solution is correct. (It is our hope that all solutions appearing here are correct; errors should be reported to the authors.) If you get stuck on an exercise, try skimming the solution to get a hint for how to proceed, but then work out the exercise yourself. The more you can do on your own, the more you will learn. Please note: for students taking a course based on *Applied Linear Algebra*, copying solutions from this Manual can place you in violation of academic honesty. In particular, many solutions here just provide the final answer, and for full credit one must also supply an explanation of how this is found.

Acknowledgements

We thank a number of people, who are named in the text, for corrections to the solutions manuals that accompanied the first edition. Of course, as authors, we take full responsibility for all errors that may yet appear. We encourage readers to inform us of any misprints, errors, and unclear explanations that they may find, and will accordingly update this manual on a timely basis. Corrections will be posted on the text's dedicated web site:

`http://www.math.umn.edu/∼olver/ala2.html`

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Chapter 1: Linear Algebraic Systems

1.1.1. (*b*) Reduce the system to $6u + v = 5$, $-\frac{5}{2}v = \frac{5}{2}$; then use Back Substitution to solve for $u = 1$, $v = -1$.

(*d*) Reduce the system to $2u - v + 2w = 2$, $-\frac{3}{2}v + 4w = 2$, $-w = 0$; then solve for $u = \frac{1}{3}$, $v = -\frac{4}{3}$, $w = 0$.

(*f*) Reduce the system to $x + z - 2w = -3$, $-y + 3w = 1$, $-4z - 16w = -4$, $6w = 6$; then solve for $x = 2$, $y = 2$, $z = -3$, $w = 1$.

♡ 1.1.3. (a) With Forward Substitution, we just start with the top equation and work down. Thus $2x = -6$ so $x = -3$. Plugging this into the second equation gives $12 + 3y = 3$, and so $y = -3$. Plugging the values of x and y in the third equation yields $-3 + 4(-3) - z = 7$, and so $z = -22$.

1.2.1. (a) 3×4, (*b*) 7, (*c*) 6, (*d*) $(\,-2 \quad 0 \quad 1 \quad 2\,)$, (*e*) $\begin{pmatrix} 0 \\ 2 \\ -6 \end{pmatrix}$.

1.2.2. Examples: (a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, (*c*) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 7 & 8 & 9 & 3 \end{pmatrix}$, (*e*) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

1.2.4. (*b*) $A = \begin{pmatrix} 6 & 1 \\ 3 & -2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} u \\ v \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$;

(*d*) $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$;

(*f*) $A = \begin{pmatrix} 1 & 0 & 1 & -2 \\ 2 & -1 & 2 & -1 \\ 0 & -6 & -4 & 2 \\ 1 & 3 & 2 & -1 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}$.

1.2.5. (*b*) $u + w = -1$, $u + v = -1$, $v + w = 2$. The solution is $u = -2$, $v = 1$, $w = 1$.

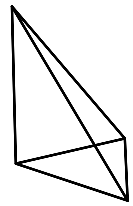
(*c*) $3x_1 - x_3 = 1$, $-2x_1 - x_2 = 0$, $x_1 + x_2 - 3x_3 = 1$.

The solution is $x_1 = \frac{1}{5}$, $x_2 = -\frac{2}{5}$, $x_3 = -\frac{2}{5}$.

1.2.6. (*a*) $\mathrm{I} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$, $\mathrm{O} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.

(*b*) $\mathrm{I} + \mathrm{O} = \mathrm{I}$, $\mathrm{I}\,\mathrm{O} = \mathrm{O}\,\mathrm{I} = \mathrm{O}$. No, it does not.

♣ 6.3.11. (a)



$$A = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix};$$

$$(b)\ \ \mathbf{v}_1 = \begin{pmatrix} 1\\0\\0\\1\\0\\0\\1\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0\\1\\0\\0\\1\\0\\0\\1\\0\\0\\1\\0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0\\0\\1\\0\\0\\1\\0\\0\\1\\0\\0\\1 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\-1\\0\\1\\0 \end{pmatrix}, \quad \mathbf{v}_5 = \begin{pmatrix} 0\\0\\0\\0\\0\\-1\\0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \mathbf{v}_6 = \begin{pmatrix} 0\\0\\0\\0\\-1\\0\\1\\0\\0\\0\\0\\0 \end{pmatrix};$$

(*c*) $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ correspond to translations in, respectively, the x, y, z directions;

(*d*) $\mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6$ correspond to rotations around, respectively, the x, y, z coordinate axes;

$$(e)\ \ K = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 2 & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix};$$

(*f*) For $\mathbf{f}_i = (f_i, g_i, h_i)^T$ we require $f_1 + f_2 + f_3 + f_4 = 0$, $g_1 + g_2 + g_3 + g_4 = 0$, $h_1 + h_2 + h_3 + h_4 = 0$, $h_3 = g_4$, $h_2 = f_4$, $g_2 = f_3$, i.e., there is no net horizontal force and no net moment of force around any axis.

(*g*) You need to fix three nodes. Fixing two still leaves a rotation motion around the line connecting them.

(*h*) Displacement of the top node: $\mathbf{u}_4 = (-1, -1, -1)^T$; since $\mathbf{e} = (-1, 0, 0, 0)^T$, only the vertical bar experiences compression of magnitude 1.

10.4.33. (b) $\begin{pmatrix} 1 & 0 & t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ — shear transformations in the x direction, with magnitude proportional to the z coordinate. The trajectories are lines parallel to the x axis. Points on the xy plane are fixed.

(d) $\begin{pmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & e^t \end{pmatrix}$ — spiral motions around the z axis. The trajectories are the positive and negative z axes, circles in the xy plane, and cylindrical spirals (helices) winding around the z axis while going away from the xy pane at an exponentially increasing rate. The only fixed point is the origin.

10.4.35.

$$\left[\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right] = \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix}, \qquad \left[\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\right] = \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix},$$
$$\left[\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix}\right] = \begin{pmatrix} 0 & -2 \\ -8 & 0 \end{pmatrix}, \qquad \left[\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix},$$

10.4.41. In the matrix system $\dfrac{dU}{dt} = AU$, the equations in the last row are $\dfrac{du_{nj}}{dt} = 0$ for $j = 1, \ldots, n$, and hence the last row of $U(t)$ is constant. In particular, for the exponential matrix solution $U(t) = e^{tA}$ the last row must equal the last row of the identity matrix $U(0) = \mathrm{I}$, which is $\mathbf{e}_n^T$.

♢ 10.4.43. (a) $\begin{pmatrix} x+t \\ y \end{pmatrix}$: translations in x direction.

(c) $\begin{pmatrix} (x+1)\cos t - y \sin t - 1 \\ (x+1)\sin t + y \cos t \end{pmatrix}$: rotations around the point $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

10.4.44. (a) If $U \neq \{\mathbf{0}\}$, then the system has eigenvalues with positive real part corresponding to exponentially growing solutions and so the origin is unstable. If $C \neq \{\mathbf{0}\}$, then the system has eigenvalues with zero real part corresponding to either bounded solutions, which are stable but not asymptotically stable modes, or, if the eigenvalue is incomplete, polynomial growing unstable modes.

10.4.45. (a) $S = U = \varnothing, \;\; C = \mathbb{R}^2$;

(b) $S = \operatorname{span}\begin{pmatrix} \frac{2-\sqrt{7}}{3} \\ 1 \end{pmatrix}, \;\; U = \operatorname{span}\begin{pmatrix} \frac{2+\sqrt{7}}{3} \\ 1 \end{pmatrix}, \;\; C = \varnothing$;

(d) $S = \operatorname{span}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \;\; U = \operatorname{span}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \;\; C = \operatorname{span}\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}.$

10.4.47. (b) $u_1(t) = e^{t-1} - e^t + t\,e^t, \;\; u_2(t) = e^{t-1} - e^t + t\,e^t$;

(d) $u(t) = \frac{13}{16} e^{4t} + \frac{3}{16} - \frac{1}{4} t, \;\; v(t) = \frac{13}{16} e^{4t} - \frac{29}{16} + \frac{3}{4} t.$