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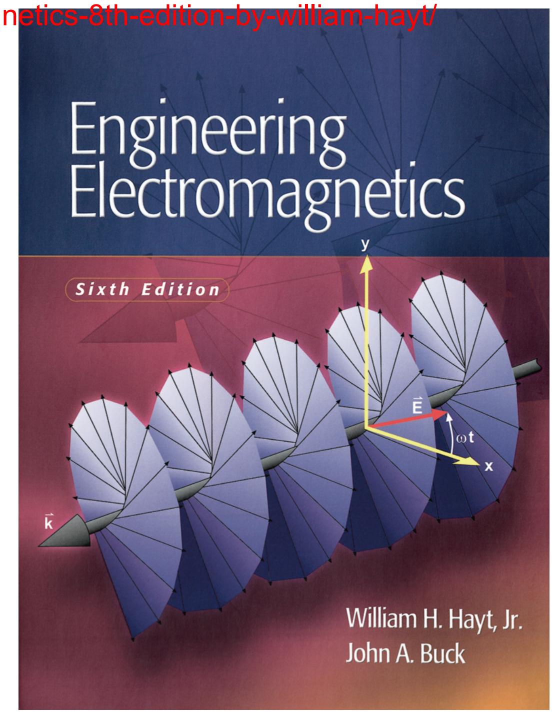

Engineering Electromagnetics

Sixth Edition

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Textbook Table of Contents

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since $J_x = 0$ there. Current will pass through the three remaining surfaces, and will be found through

$$I = \int_2^3 \int_0^1 \mathbf{J} \cdot (-\mathbf{a}_y)\Big|_{y=0} dx\,dz + \int_2^3 \int_0^1 \mathbf{J} \cdot (\mathbf{a}_y)\Big|_{y=1} dx\,dz + \int_2^3 \int_0^1 \mathbf{J} \cdot (\mathbf{a}_x)\Big|_{x=1} dy\,dz$$
$$= 10^4 \int_2^3 \int_0^1 \left[\cos(2x)e^{-0} - \cos(2x)e^{-2}\right] dx\,dz - 10^4 \int_2^3 \int_0^1 \sin(2)e^{-2y}\,dy\,dz$$
$$= 10^4 \left(\frac{1}{2}\right) \sin(2x)\Big|_0^1 (3-2)\left[1 - e^{-2}\right] + 10^4 \left(\frac{1}{2}\right) \sin(2)e^{-2y}\Big|_0^1 (3-2) = \underline{0}$$

c) Repeat part b, but use the divergence theorem: We find the net outward current through the surface of the cube by integrating the divergence of $\mathbf{J}$ over the cube volume. We have

$$\nabla \cdot \mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = -10^{-4}\left[2\cos(2x)e^{-2y} - 2\cos(2x)e^{-2y}\right] = \underline{0} \text{ as expected}$$

5.11. Two perfectly-conducting cylindrical surfaces are located at $\rho = 3$ and $\rho = 5$ cm. The total current passing radially outward through the medium between the cylinders is 3 A dc. *Assume the cylinders are both of length l.*

a) Find the voltage and resistance between the cylinders, and $\mathbf{E}$ in the region between the cylinders, if a conducting material having $\sigma = 0.05\,\text{S/m}$ is present for $3 < \rho < 5$ cm: Given the current, and knowing that it is radially-directed, we find the current density by dividing it by the area of a cylinder of radius ρ and length l:

$$\mathbf{J} = \frac{3}{2\pi\rho l}\,\mathbf{a}_\rho \text{ A/m}^2$$

Then the electric field is found by dividing this result by σ:

$$\mathbf{E} = \frac{3}{2\pi\sigma\rho l}\,\mathbf{a}_\rho = \underline{\frac{9.55}{\rho l}\mathbf{a}_\rho \text{ V/m}}$$

The voltage between cylinders is now:

$$V = -\int_5^3 \mathbf{E} \cdot \mathbf{dL} = \int_3^5 \frac{9.55}{\rho l}\mathbf{a}_\rho \cdot \mathbf{a}_\rho d\rho = \frac{9.55}{l} \ln\left(\frac{5}{3}\right) = \underline{\frac{4.88}{l} \text{ V}}$$

Now, the resistance will be

$$R = \frac{V}{I} = \frac{4.88}{3l} = \underline{\frac{1.63}{l}\,\Omega}$$

b) Show that integrating the power dissipated per unit volume over the volume gives the total dissipated power: We calculate

$$P = \int_v \mathbf{E} \cdot \mathbf{J}\,dv = \int_0^l \int_0^{2\pi} \int_{.03}^{.05} \frac{3^2}{(2\pi)^2\rho^2(.05)l^2}\,\rho\,d\rho\,d\phi\,dz = \frac{3^2}{2\pi(.05)l} \ln\left(\frac{5}{3}\right) = \frac{14.64}{l} \text{ W}$$

We also find the power by taking the product of voltage and current:

$$P = VI = \frac{4.88}{l}(3) = \frac{14.64}{l} \text{ W}$$

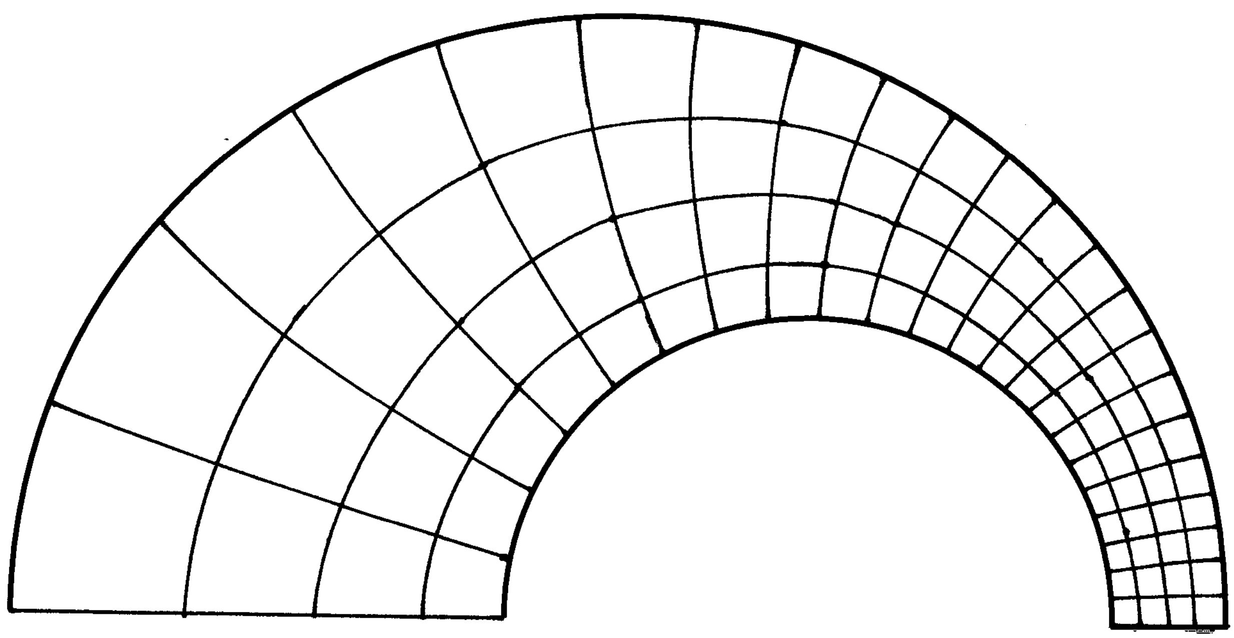
6.3. Construct a curvilinear square map of the potential field between two parallel circular cylinders, one of 4-cm radius inside one of 8-cm radius. The two axes are displaced by 2.5 cm. These dimensions are suitable for the drawing. As a check on the accuracy, compute the capacitance per meter from the sketch and from the exact expression:

$$C = \frac{2\pi\epsilon}{\cosh^{-1}\left[(a^2+b^2-D^2)/(2ab)\right]}$$

where a and b are the conductor radii and D is the axis separation.

Our attempt at the drawing is shown below. Use of the exact expression above yields a capacitance value of $C = \underline{11.5\epsilon_0 \text{ F/m}}$. Use of the drawing produces:

$$C \doteq \frac{22 \times 2}{4}\epsilon_0 = \underline{11\epsilon_0 \text{ F/m}}$$



10.17. (continued) Now

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon_0} = \int \frac{1}{\epsilon_0} \nabla \times \mathbf{H}\, dt + C_2 = \frac{C(144+a^2)}{\mu_0\epsilon_0(2\times 10^{10})^2} \sin(12y)\sin(az)\cos(2\times 10^{10}t)\,\mathbf{a}_x$$

where $C_2 = 0$. This field must be the same as the original field as stated, and so we require that

$$\frac{C(144+a^2)}{\mu_0\epsilon_0(2\times 10^{10})^2} = 1$$

Using $\mu_0\epsilon_0 = (3\times 10^8)^{-2}$, we find

$$a = \left[\frac{(2\times 10^{10})^2}{(3\times 10^8)^2} - 144\right]^{1/2} = \underline{66}$$

11.7. The phasor magnetic field intensity for a 400-MHz uniform plane wave propagating in a certain lossless material is $(2\mathbf{a}_y - j5\mathbf{a}_z)e^{-j25x}$ A/m. Knowing that the maximum amplitude of $\mathbf{E}$ is 1500 V/m, find β, η, λ, v_p, ϵ_R, μ_R, and $\mathbf{H}(x,y,z,t)$: First, from the phasor expression, we identify $\beta = \underline{25\ \text{m}^{-1}}$ from the argument of the exponential function. Next, we evaluate $H_0 = |\mathbf{H}| = \sqrt{\mathbf{H}\cdot\mathbf{H}^*} = \sqrt{2^2+5^2} = \sqrt{29}$. Then $\eta = E_0/H_0 = 1500/\sqrt{29} = \underline{278.5\ \Omega}$. Then $\lambda = 2\pi/\beta = 2\pi/25 = .25$ m $= \underline{25\text{ cm}}$. Next,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi\times 400\times 10^6}{25} = \underline{1.01\times 10^8\ \text{m/s}}$$

Now we note that

$$\eta = 278.5 = 377\sqrt{\frac{\mu_R}{\epsilon_R}} \quad\Rightarrow\quad \frac{\mu_R}{\epsilon_R} = 0.546$$

And

$$v_p = 1.01\times 10^8 = \frac{c}{\sqrt{\mu_R\epsilon_R}} \quad\Rightarrow\quad \mu_R\epsilon_R = 8.79$$

We solve the above two equations simultaneously to find $\epsilon_R = \underline{4.01}$ and $\mu_R = \underline{2.19}$. Finally,

$$\begin{aligned}\mathbf{H}(x,y,z,t) &= Re\left\{(2\mathbf{a}_y - j5\mathbf{a}_z)e^{-j25x}e^{j\omega t}\right\}\\ &= \underline{2\cos(2\pi\times 400\times 10^6 t - 25x)\mathbf{a}_y + 5\sin(2\pi\times 400\times 10^6 t - 25x)\mathbf{a}_z\ \text{A/m}}\end{aligned}$$

11.15. A 10 GHz radar signal may be represented as a uniform plane wave in a sufficiently small region. Calculate the wavelength in centimeters and the attenuation in nepers per meter if the wave is propagating in a non-magnetic material for which

a) $\epsilon_R' = 1$ and $\epsilon_R'' = 0$: In a non-magnetic material, we would have:

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon_R'}{2}}\left[\sqrt{1+\left(\frac{\epsilon_R''}{\epsilon_R'}\right)^2} - 1\right]^{1/2}$$

and

$$\beta = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon_R'}{2}}\left[\sqrt{1+\left(\frac{\epsilon_R''}{\epsilon_R'}\right)^2} + 1\right]^{1/2}$$

With the given values of ϵ_R' and ϵ_R'', it is clear that $\beta = \omega\sqrt{\mu_0\epsilon_0} = \omega/c$, and so $\lambda = 2\pi/\beta = 2\pi c/\omega = 3\times 10^{10}/10^{10} = \underline{3\text{ cm}}$. It is also clear that $\alpha = \underline{0}$.