SOLUTIONS MANUAL

MECHANICAL VIBRATIONS

FIFTH EDITION

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To Lord Sri Venkateswara

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direction of x, k_e, is given by Eq. (6):

$$k_e = \frac{F}{x} = \frac{(l_1 + l_2)^2 \, k_1 k_2}{l_1^2 k_1 + l_2^2 k_2} \qquad (7)$$

1.58 $c = \mu \left\{ \frac{3 \pi D^3 l}{4 d^3} \left(1 + 2 \frac{d}{D}\right) \right\}$;

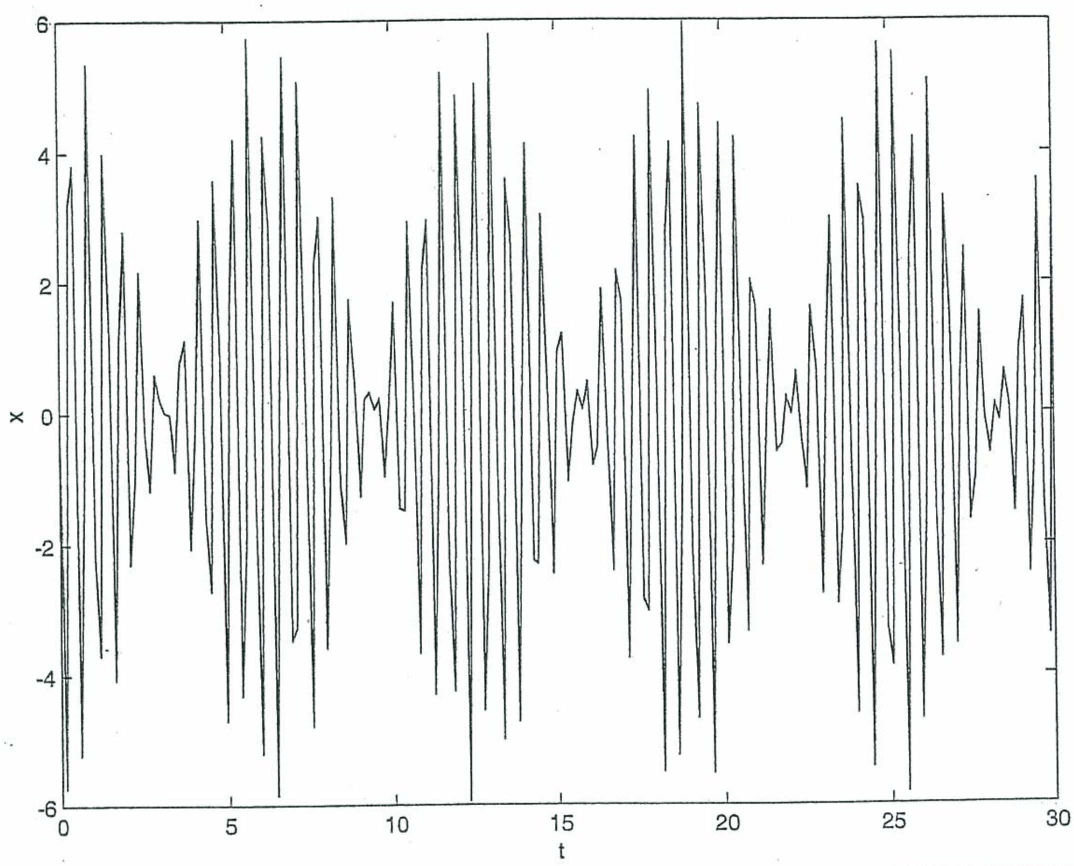
D = diameter of piston
l = axial length of piston
d = radial clearance

$\mu = 45$ μ reynolds
(from Shigley's Mechanical Engineering Design)

Let $d = 0.001''$, $D = 2.4''$ and above equation gives

$$10^5 = (45 \times 10^{-6}) \left\{ \frac{3 \pi (2.4)^3 l}{4 (0.001)^3} \left(1 + \frac{2 \times 0.001}{2.4}\right) \right\}$$

$\therefore$ $l = 0.6817''$

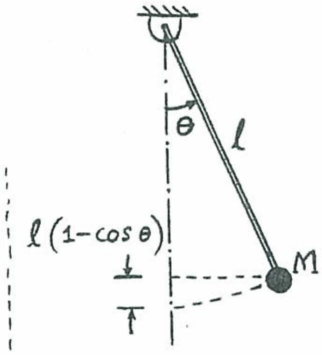

6
4
2
x 0
-2
-4
-6
0
5
10
15
20
25
30
t

2.71 Kinetic energy of system is

$$T = T_{rod} + T_{bob} = \frac{1}{2}\left(\frac{1}{3} m l^2\right) \dot{\theta}^2 + \frac{1}{2} M l^2 \dot{\theta}^2$$

Potential energy of system is
(since mass of the rod acts through its center)

$$U = U_{rod} + U_{bob} = \frac{1}{2} mgl(1-\cos\theta) + \frac{1}{2} Mgl(1-\cos\theta)$$



Equation of motion:

$$\frac{d}{dt}(T+U) = 0$$

i.e. $\left(M + \frac{m}{3}\right) l^2 \ddot{\theta} + \left(M + \frac{m}{2}\right) g l \sin\theta = 0$

For small angles, $\ddot{\theta} + \frac{\left(M+\frac{m}{2}\right)}{\left(M+\frac{m}{3}\right)} \frac{g}{l} \cdot \theta = 0$

$$\omega_n = \sqrt{\frac{\left(M+\frac{m}{2}\right) g}{\left(M+\frac{m}{3}\right) l}}$$

2.72 For the shaft, $J = \frac{\pi d^4}{32} = \frac{\pi (0.05)^4}{32} = 61.3594 \times 10^{-8}\ m^4$

$$k_t = \frac{GJ}{l} = \frac{(0.793 \times 10^{11})(61.3594 \times 10^{-8})}{2} = 24329.002\ \text{N-m/rad}$$

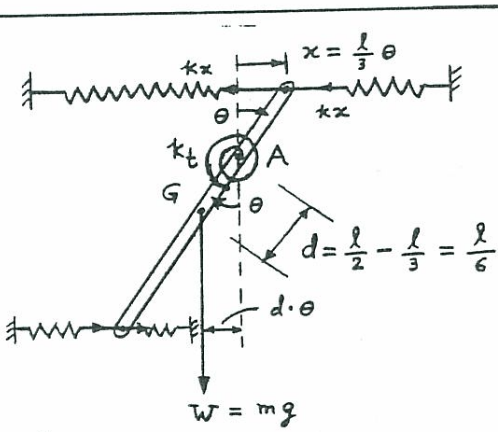
For the disc, $J_0 = \frac{MD^2}{8} = \left(\rho \frac{\pi D^2}{4} h\right)\frac{D^2}{8} = \frac{\rho \pi D^4 h}{32}$

$$= \frac{(7.83 \times 10^3)\ \pi\ (1)^4 (0.1)}{32} = 76.8710\ \text{kg-m}^2$$

$$\omega_n = \sqrt{\frac{k_t}{J_0}} = \left(\frac{24329.002}{76.8710}\right)^{1/2} = 17.7902\ \text{rad/sec}$$

2.73 Equation of motion

$$J_A \ddot{\theta} = -W d\theta - 2k\left(\frac{l}{3}\theta\right)\frac{l}{3} - 2k\left(\frac{2l}{3}\theta\right)\frac{2l}{3} - k_t \theta$$



where

$$J_A = J_G + m d^2 = \frac{1}{12} m l^2 + m \frac{l^2}{36}$$

$$= \frac{1}{9} m l^2$$

$$\therefore \frac{m l^2}{9} \ddot{\theta} + \left(mgd + \frac{2kl^2}{9} + \frac{8kl^2}{9} + k_t\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{\left(mgd + \frac{2}{9}kl^2 + \frac{8}{9}kl^2 + k_t\right)9}{m l^2}} = \sqrt{\frac{9mgd + 10kl^2 + 9k_t}{m l^2}}$$

```
      XM=1.0
      DATA F/1.0,.8436,.6910,.5460,.4122,.2929,.1910,.1090,
     2 .04894,.01231,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0/
      DO 10 I=1,21
 10   DELT(I)=0.31416
C END OF PROBLEM-DEPENDENT DATA
      A(1)=0.0
      B(1)=0.0
      OMD=OMN*SQRT(1.0-XAI**2)
      T(1)=0.0
      DO 20 I=2,NP
      T(I)=T(I-1)+DELT(I)
      TIME=T(I)
      CALL PI1(TIME,XAI,OMN,OMD,PP1)
      CALL PI2(TIME,XAI,OMN,OMD,PP2)
      CALL PI3(TIME,XAI,OMN,OMD,PP3)
      CALL PI4(TIME,XAI,OMN,OMD,PP4)
      TIME=T(I-1)
      CALL PI1(TIME,XAI,OMN,OMD,PM1)
      CALL PI2(TIME,XAI,OMN,OMD,PM2)
      CALL PI3(TIME,XAI,OMN,OMD,PM3)
      CALL PI4(TIME,XAI,OMN,OMD,PM4)
      P1=PP1-PM1
      P2=PP2-PM2
      P3=PP3-PM3
      P4=PP4-PM4
      DELF=F(I)-F(I-1)
      A(I)=A(I-1)+(DELF/DELT(I))*P1+(F(I-1)-T(I-1)*DELF/DELT(I))*P2
      B(I)=B(I-1)+(DELF/DELT(I))*P4+(F(I-1)-T(I-1)*DELF/DELT(I))*P3
      X(I)=(EXP(-XAI*OMN*T(I))/(XM*OMD))*(A(I)*SIN(OMD*T(I))-
     2 B(I)*COS(OMD*T(I)))
 20   CONTINUE
      PRINT 30
 30   FORMAT (//,2X,41H NUMERICAL EVALUATION OF DUHAMEL INTEGRAL,
     2 //,5X,2H I,6X,5H T(I),10X,5H F(I),10X,5H X(I),/)
      DO 40 I=2,NP
 40   PRINT 50, I,T(I),F(I),X(I)
 50   FORMAT (2X,I5,3E15.8)
      STOP
      END
C ====================================================================
C
C SUBROUTINE PI1
C
C ====================================================================
      SUBROUTINE PI1 (T,XAI,OMN,OMD,P)
      DEN=(XAI*OMN)**2+OMD**2
      P=(T*EXP(XAI*OMN*T)/DEN)*(XAI*OMN*COS(OMD*T)+OMD*SIN(OMD*T))
     2 -(EXP(XAI*OMN*T)/(DEN**2))*(((XAI*OMN)**2-OMD**2)*COS(OMD*T)
     3 +2.0*XAI*OMN*OMD*SIN(OMD*T))
      RETURN
      END
```

(b) New system:

Natural frequencies of the new system, Ω_1 and Ω_2, are given by (for $\omega_2/\omega_1 = 1$):

$$r_1^2,\ r_2^2 = \left(1+\frac{\mu}{2}\right) \mp \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}$$

where $r_1 = \Omega_1/\omega_2$, $r_2 = \Omega_2/\omega_2$ and $\mu = m_2/m_1$.

Here $\mu = 1.2665/5.176 = 0.2447$, and hence

$r_1^2 = 0.6127$, $\quad r_1 = 0.7827$

$r_2^2 = 1.6319$, $\quad r_2 = 1.2775$

$\Omega_1 = r_1\,\omega_2 = 0.7827\,(62.832) = 49.1818$ rad/sec $= 469.6509$ rpm

$\Omega_2 = r_2\,\omega_2 = 1.2775\,(62.832) = 80.2653$ rad/sec $= 766.4750$ rpm

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9.67 Natural frequencies of the combined system are

$\Omega_1 = 0.7\,\omega_2 = 0.7\,(62.832) = 43.9824$ rad/sec

$\Omega_2 = 1.3\,\omega_2 = 1.3\,(62.832) = 81.6816$ rad/sec

$r_1 = 0.7$, $r_2 = 1.3$, $\sqrt{k_2/m_2} = \omega_2 = \omega_1 = 62.832$ rad/sec

(for tuned absorber)

$k_2 = m_2\,\omega_2^2 = (62.832)^2\, m_2 \quad \ldots \ (E_1)$

$$r_1^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} \qquad \text{or} \qquad \mu = \frac{r_1^4+1}{r_1^2} - 2$$

$$\mu = \frac{(0.7)^4+1}{(0.7)^2} - 2 = 0.5308 = \frac{m_2}{m_1} \Rightarrow m_2 = 0.5308\,(5.176) = 2.7474 \text{ lb-sec}^2/\text{in.}$$

From Eq. (E_1), $\quad k_2 = (62.832)^2\,(2.7474) = 10846.3512$ lb/in.

Verification of r_2:

$$r_2^2 = \left(1+\frac{\mu}{2}\right) + \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} = \left(1+\frac{0.5308}{2}\right) + \sqrt{\left(1+\frac{0.5308}{2}\right)^2 - 1}$$

$= 2.0408$

or $\quad r_2 = 1.4286 \quad > 1.3$ (desired value)

Hence: $\quad m_2 = 2.7474$ lb-sec^2/in

(weight of absorber $= m_2 g = 1061.5954$ lb)

$k_2 = 10\,846.3512$ lb/in.
