Chapter 1 • Introduction

1.1 A gas at 20°C may be *rarefied* if it contains less than 10^{12} molecules per mm^3. If Avogadro's number is 6.023E23 molecules per mole, what air pressure does this represent?

Solution: The mass of one molecule of air may be computed as

$$m = \frac{\text{Molecular weight}}{\text{Avogadro's number}} = \frac{28.97 \text{ mol}^{-1}}{6.023\text{E23 molecules/g} \cdot \text{mol}} = 4.81\text{E–23 g}$$

Then the density of air containing 10^{12} molecules per mm^3 is, in SI units,

$$\rho = \left(10^{12} \ \frac{\text{molecules}}{\text{mm}^3}\right)\left(4.81\text{E–23} \ \frac{\text{g}}{\text{molecule}}\right)$$

$$= 4.81\text{E–11} \ \frac{\text{g}}{\text{mm}^3} = 4.81\text{E–5} \ \frac{\text{kg}}{\text{m}^3}$$

Finally, from the perfect gas law, Eq. (1.13), at 20°C = 293 K, we obtain the pressure:

$$p = \rho RT = \left(4.81\text{E–5} \ \frac{\text{kg}}{\text{m}^3}\right)\left(287 \ \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}\right)(293 \text{ K}) = \textbf{4.0 Pa} \quad Ans.$$

1.2 The earth's atmosphere can be modeled as a uniform layer of air of thickness 20 km and average density 0.6 kg/m^3 (see Table A-6). Use these values to estimate the total mass and total number of molecules of air in the entire atmosphere of the earth.

Solution: Let R_e be the earth's radius $\approx$ 6377 km. Then the total mass of air in the atmosphere is

$$m_t = \int \rho \, d\text{Vol} = \rho_{avg}(\text{Air Vol}) \approx \rho_{avg} 4\pi R_e^2(\text{Air thickness})$$

$$= (0.6 \text{ kg/m}^3)4\pi(6.377\text{E6 m})^2(20\text{E3 m}) \approx \textbf{6.1E18 kg} \quad Ans.$$

Dividing by the mass of one molecule $\approx$ 4.8E–23 g (see Prob. 1.1 above), we obtain the total number of molecules in the earth's atmosphere:

$$N_{molecules} = \frac{m(\text{atmosphere})}{m(\text{one molecule})} = \frac{6.1\text{E21 grams}}{4.8\text{E–23 gm/molecule}} \approx \textbf{1.3E44 molecules} \quad Ans.$$

1.3   For the triangular element in Fig. P1.3, show that a tilted free liquid surface, in contact with an atmosphere at pressure p_a, must undergo shear stress and hence begin to flow.
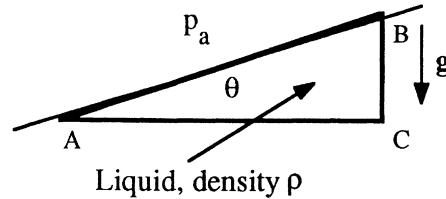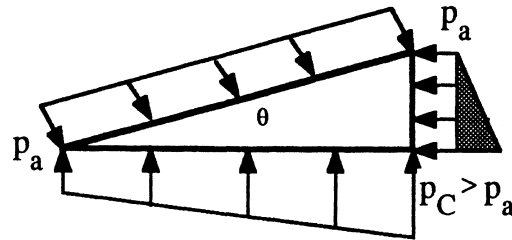

Fig. P1.3

Solution: Assume zero shear. Due to element weight, the pressure along the lower and right sides must vary linearly as shown, to a higher value at point C. Vertical forces are presumably in balance with element weight included. But horizontal forces are out of balance, with the unbalanced force being to the left, due to the shaded excess-pressure triangle on the right side BC. Thus hydrostatic pressures cannot keep the element in balance, and shear and flow result.

1.4   The quantities viscosity μ, velocity V, and surface tension Y may be combined into a dimensionless group. Find the combination which is proportional to μ. This group has a customary name, which begins with C. Can you guess its name?

Solution: The dimensions of these variables are $\{\mu\} = \{M/LT\}$, $\{V\} = \{L/T\}$, and $\{Y\} = \{M/T^2\}$. We must divide μ by Y to cancel mass $\{M\}$, then work the velocity into the group:

$$\left\{\frac{\mu}{Y}\right\} = \left\{\frac{M/LT}{M/T^2}\right\} = \left\{\frac{T}{L}\right\}, \quad \textit{hence multiply by } \{V\} = \left\{\frac{L}{T}\right\};$$

$$\textit{finally obtain } \frac{\mu V}{Y} = \textit{\textbf{dimensionless.}} \quad \textit{Ans.}$$

This dimensionless parameter is commonly called the *Capillary Number*.

1.5   A formula for estimating the mean free path of a perfect gas is:

$$\ell = 1.26 \frac{\mu}{\rho\sqrt{(RT)}} = 1.26 \frac{\mu}{p}\sqrt{(RT)} \tag{1}$$

2.80   For the closed tank of Fig. P2.80, all fluids are at 20°C and the air space is pressurized. If the outward net hydrostatic force on the 40-cm by 30-cm panel at the bottom is 8450 N, estimate (a) the pressure in the air space; and (b) the reading h on the manometer.
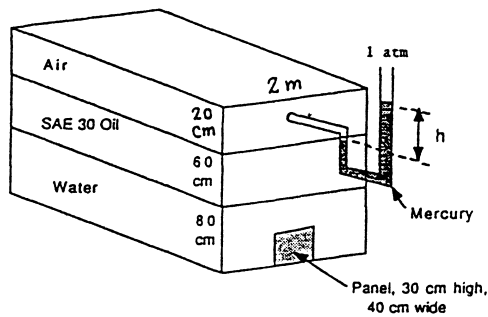
Fig. P2.80

Solution: The force on the panel yields water (gage) pressure at the centroid of the panel:

$$F = 8450 \text{ N} = p_{CG}A = p_{CG}(0.3 \times 0.4 \text{ m}^2), \quad \text{or} \quad p_{CG} = 70417 \text{ Pa (gage)}$$

This is the water pressure 15 cm above the bottom. Now work your way back through the two liquids to the air space:

$$p_{air\ space} = 70417 \text{ Pa} - (9790)(0.80 - 0.15) - 8720(0.60) = \mathbf{58800\ Pa} \quad \textit{Ans.} \text{ (a)}$$

Neglecting the specific weight of air, we move out through the mercury to the atmosphere:

$$58800 \text{ Pa} - (133100 \text{ N/m}^3)h = p_{atm} = 0 \text{ (gage)}, \quad \text{or:} \quad h = \mathbf{0.44\ m} \quad \textit{Ans.} \text{ (b)}$$

2.81   Gate AB is 7 ft into the paper and weighs 3000 lbf when submerged. It is hinged at B and rests against a smooth wall at A. Find the water level h which will just cause the gate to open.
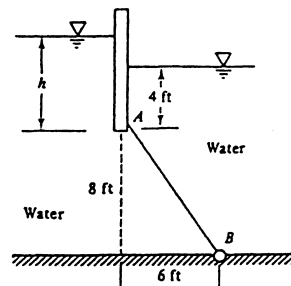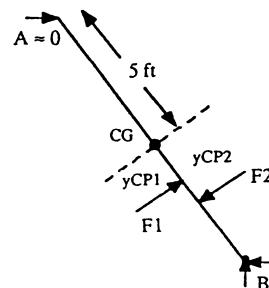
Solution: On the right side, $h_{CG} = 8$ ft, and

$$F_2 = \gamma h_{CG2}A_2$$
$$= (62.4)(8)(70) = 34944 \text{ lbf}$$

$$y_{CP2} = -\frac{(1/12)(7)(10)^3 \sin(53.13°)}{(8)(70)}$$
$$= -0.833 \text{ ft}$$

Fig. P2.81

3.183 The pump in Fig. P3.183 draws gasoline at 20°C from a reservoir. Pumps are in big trouble if the liquid vaporizes (cavitates) before it enters the pump. (a) Neglecting losses and assuming a flow rate of 65 gal/min, find the limitations on (x, y, z) for avoiding cavitation. (b) If pipe-friction losses are included, what additional limitations might be important?

Solution: (a) From Table A.3, $\rho = 680$ kg/m^3 and $p_v = 5.51E+4$.

$$z_2 - z_1 = y + z = \frac{p_1 - p_2}{\rho g} = \frac{(p_a + \rho g y) - p_v}{\rho g}$$

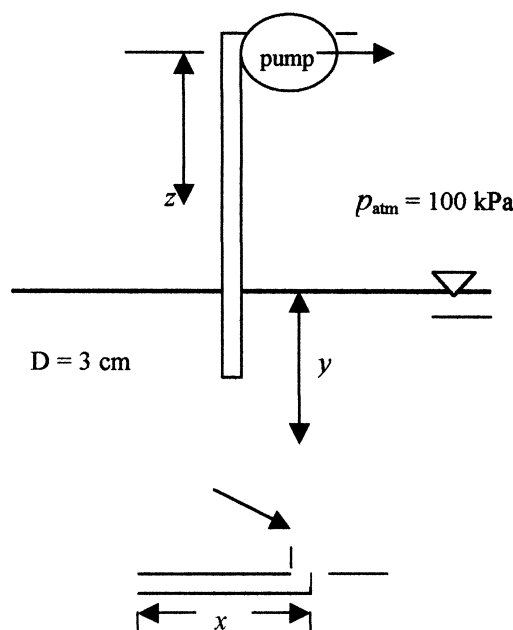$$y + z = \frac{(100,000 - 55,100)}{(680)(9.81)} + y \quad z = 6.73 \ m$$

Fig. P3.183

$p_{atm} = 100$ kPa

$D = 3$ cm

Thus make length z appreciably less than 6.73 (25% less), or **z < 5 m**.   *Ans.* (a)

(b) **Total pipe length $(x + y + z)$ restricted by friction losses**.   *Ans.* (b)

3.184 For the system of Prob. 3.183, let the pump exhaust gasoline at 65 gal/min to the atmosphere through a 3-cm-diameter opening, with no cavitation, when $x = 3$ m, $y = 2.5$ m, and $z = 2$ m. If the friction head loss is $h_{loss} \approx 3.7(V^2/2g)$, where V is the average velocity in the pipe, estimate the horsepower required to be delivered by the pump.

Solution: Since power is a function of h_p, Bernoulli is required. Thus calculate the velocity,

$$V = \frac{Q}{A} = \frac{(65 \ \text{gal/min})\left(6.3083E{-}5\dfrac{\text{m}^3/\text{s}}{\text{gal/min}}\right)}{\dfrac{\pi}{4}(0.03^2)} = 5.8 \ \text{m/s}$$

The pump head may then be found,

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_f - h_p + \frac{V_j^2}{2g}$$
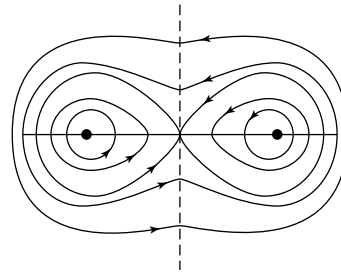
$$\frac{100,000 + (680)(9.81)(2.5)}{(680)(9.81)} - 2.5 = \frac{100,000}{(680)(9.81)} + 2 + \frac{3.7(5.8^2)}{2(9.81)} - h_p + \frac{(5.8^2)}{2(9.81)}$$

4.69   Find the stream function and plot some streamlines for the combination of a counter-clockwise line vortex K at $(x, y) = (+a, 0)$ and an equal line vortex placed at $(-a, 0)$.

Solution: The combined stream function is

$$\psi = -K \ln r_1 - K \ln r_2 = -K \ln[(x-a)^2 + y^2]^{1/2}$$
$$- K \ln[(x+a)^2 + y^2]^{1/2}$$

Plotting this, for various $K = $ constant, reveals the "cat's-eye" pattern shown at right.

4.70   Take the limit of ϕ for the source-sink combination, Eq. (4.133), as strength m becomes large and distance a becomes small, so that $(ma) = $ constant. What happens?

Solution: Given $\phi = \frac{1}{2}m \ln[\{(x+a)^2 + y^2\}/\{(x-a)^2 + y^2\}]$, divide [] by $(x+a)^2$ and use the series form $\ln[(1+\varepsilon)/(1-\varepsilon)] = 2\varepsilon + 2\varepsilon^2/3 + \cdots$ the result is the *line doublet*:

$$\phi_{doublet} = \mathbf{lim}|_{am=0} \ (\phi_{source+sink}) = \frac{2amx}{x^2 + y^2} = \frac{\lambda \cos\theta}{r^2}, \quad \lambda = 2am \quad Ans.$$

4.71   Find the stream function and plot some streamlines for the combination of a counterclockwise line vortex K at $(x, y) = (+a, 0)$ and an opposite (clockwise) line vortex placed at $(-a, 0)$.

Solution: The combined stream function is

$$\psi = -K \ln r_1 + K \ln r_2 = -K \ln[(x-a)^2 + y^2]^{1/2}$$
$$+ K \ln[(x+a)^2 + y^2]^{1/2}$$

Plotting this, for various $K = $ constant, reveals the swirling "vortex-pair" pattern shown at right. It is equivalent to an "image" vortex pattern, as in Fig. 8.17(b) of the text.
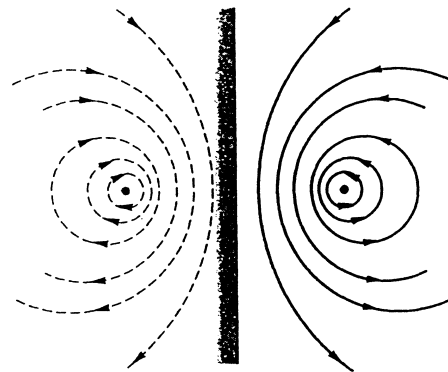
Fig. P4.71

Solve this problem when $h = 58$ cm is known and Q is the unknown. Well, we can see that the numbers are the same as part (a), and the solution is

$$\text{Solve for:} \quad Q \approx 0.00556 \text{ m}^3/\text{s} = \mathbf{20 \text{ m}^3/h} \quad \textit{Ans.} \text{ (b)}$$

6.155   It is desired to meter a flow of 20°C gasoline in a 12-cm-diameter pipe, using a modern venturi nozzle. In order for international standards to be valid (Fig. 6.40), what is the permissible range of (a) flow rates, (b) nozzle diameters, and (c) pressure drops? (d) For the highest pressure-drop condition, would compressibility be a problem?

Solution: For gasoline at 20°C, take $\rho = 680$ kg/m^3 and $\mu = 2.92E–4$ kg/m·s. Examine the possible range of Reynolds number and beta ratio:

$$1.5E5 < Re_D = \frac{4\rho Q}{\pi\mu D} = \frac{4(680)Q}{\pi(2.92E{-}4)(0.12)} < 2.0E5,$$

$$\text{or} \quad \mathbf{0.0061 < Q < 0.0081} \ \frac{\mathbf{m}^3}{\mathbf{s}} \quad \textit{Ans.} \text{ (a)}$$

$$0.316 < \beta = d/D < 0.775, \quad \text{or:} \quad \mathbf{3.8 < d < 9.3 \text{ cm}} \quad \textit{Ans.} \text{ (b)}$$

For estimating pressure drop, first compute $C_d(\beta)$ from Eq. (6.116): $0.924 < C_d < 0.985$:

$$Q = C_d \frac{\pi}{4}(0.12\beta)^2 \sqrt{\frac{2\Delta p}{680(1-\beta^4)}}, \quad \text{or:} \quad \Delta p = 2.66E6(1-\beta^4)\left[\frac{Q}{C_d\beta^2}\right]^2$$

put in large Q, small β, etc. to obtain the range $\mathbf{200 < \Delta p < 18000 \text{ Pa}}$ $\quad$ *Ans.* (c)

6.156   Ethanol at 20°C flows down through a modern venturi nozzle as in Fig. P6.156. If the mercury manometer reading is 4 in, as shown, estimate the flow rate, in gal/min.

Solution: For ethanol at 20°C, take $\rho = 1.53$ slug/ft^3 and $\mu = 2.51E{-}5$ slug/ft·s. Given $\beta = 0.5$, the discharge coefficient is

$$C_d = 0.9858 - 0.196(0.5)^{4.5} \approx 0.9771$$

$D = 6$ in

9 in

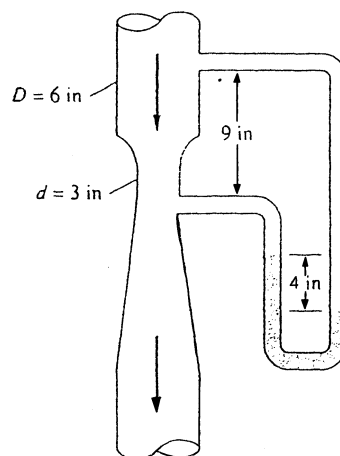$d = 3$ in

4 in

Fig. P6.156

Solution: For methane (CH_4), from Table A.4, take $k = 1.32$ and $R = 518$ J/kg·K. Tough calculation, no appendix tables for methane, should probably use EES. Find inlet density, velocity, Mach number:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{600000\,\text{Pa}}{(518\,\text{J/kg·K})(373\,\text{K})} = 3.11\,\frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = 5\,\text{kg/s} = \rho_1 A_1 V_1 = (3.11\,\text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.15\,\text{m})^2 V_1, \quad \text{solve for }\ V_1 = 91.1\,\text{m/s}$$

$$a_1 = \sqrt{kRT_1} = \sqrt{1.32(518)(373)} = 505\,\text{m/s}, \quad Ma_1 = \frac{V_1}{a_1} = \frac{91.1\,\text{m/s}}{505\,\text{m/s}} = 0.180$$

Now we have to work out the pipe-friction relations, Eqs. (9.66) and (9.68), for $k = 1.32$. We need fL^*/D, V/V^*, and p/p^* at the inlet, Ma = 0.18:

$$\frac{fL^*}{D} = \frac{1 - Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right] = 19.63 \quad \text{at}\ Ma_1 = 0.18 \quad \text{and}\quad k = 1.32$$

$$\text{Solve}\quad L^*_{choking} = 19.63\frac{D}{f} = 19.63\left(\frac{0.15\,m}{0.023}\right) = \mathbf{128\,m}\quad \textit{Ans. (a)}$$

$$\frac{fL^*}{D} = \frac{1 - Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right] = 19.63 \quad \text{at}\ Ma_1 = 0.18 \quad \text{and}\quad k = 1.32$$

$$\text{Solve}\quad L^*_{choking} = 19.63\frac{D}{f} = 19.63\left(\frac{0.15\,m}{0.023}\right) = \mathbf{128\,m}\quad \textit{Ans. (a)}$$

$$\frac{p}{p^*} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]^{1/2} = 5.954 \quad \text{at}\ Ma_1 = 0.18 \quad \text{and}\quad k = 1.32$$

$$\text{Decrease 50\% to:}\quad p/p^* = 2.977\quad \text{Solve for:}\ Ma_2 = 0.358,\ fL^*/D = 3.46$$

$$\text{Solve:}\quad \Delta L^* = (19.63 - 3.46)\frac{D}{f} = 16.17\left(\frac{0.15\,\text{m}}{0.023}\right) = \mathbf{105\,m}\quad \textit{Ans. (c)}$$

9.97   By making a few algebraic substitutions, show that Eq. (9.74), or the relation in Prob. 9.96, may be written in the density form

$$\rho_1^2 = \rho_2^2 + \rho^{*2}\left(\frac{2k}{k+1}\frac{\bar{f}L}{D} + 2\ln\frac{\rho_1}{\rho_2}\right)$$

Why is this formula awkward if one is trying to solve for the mass flow when the pressures are given at sections 1 and 2?

Apply this to the special case $b_1 = 3$ m, $b_2 = 2$ m, and $y_1 = 1.9$ m. Find the flow rate (a) if $y_2 = 1.5$ m; and (b) find the depth y_2 for which the flow becomes critical in the throat.

Solution: Given the water depths, continuity and energy allow us to eliminate one velocity:

$$\text{Continuity: } Q = V_1 y_1 b_1 = V_2 y_2 b_2; \quad \text{Energy: } y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

Eliminate V_1 to obtain $V_2 = [2g(y_1 - y_2)/(1-\alpha^2)]^{1/2}$ where $\alpha = (y_2 b_2)/(y_1 b_1)$

or: $$Q = V_2 y_2 b_2 = \left[2g(y_1 - y_2)/\left\{b_2^{-2} y_2^{-2} - b_1^{-2} y_1^{-2}\right\}\right]^{1/2} \quad Ans.$$

Evaluate the solution we just found:

$$Q = \left[\frac{2(9.81)(1.9 - 1.5)}{(2)^{-2}(1.5)^{-2} - (3)^{-2}(1.9)^{-2}}\right]^{1/2} \approx \mathbf{9.88} \ \frac{\mathbf{m^3}}{\mathbf{s}} \quad Ans. \text{ (a)}$$

For this part (a), $Fr_2 = V_2/\sqrt{(gy_2)} \approx \mathbf{0.86}$.

(b) To find critical flow, keep reducing y_2 until $Fr_2 = 1.0$. This converges to $\mathbf{y_2 \approx 1.372}$ **m**. [for which $Q = \mathbf{10.1}$ m^3/s] *Ans.* (b)

10.114 Investigate the possibility of *choking* in the venturi flume of Fig. P10.113. Let $b_1 = 4$ ft, $b_2 = 3$ ft, and $y_1 = 2$ ft. Compute the values of y_2 and V_1 for a flow rate of (a) 30 ft^3/s and (b) 35 ft^3/s. Explain your vexation.

Solution: You can't get anywhere near either $Q = 30$ or $Q = 35$ m^3/s, the flume **chokes** (becomes critical in the throat) at about $Q = 17.05$ m^3/s, when $\mathbf{y_2 \approx 1.49}$ **m**, as shown in the graph below. **Ans**.