

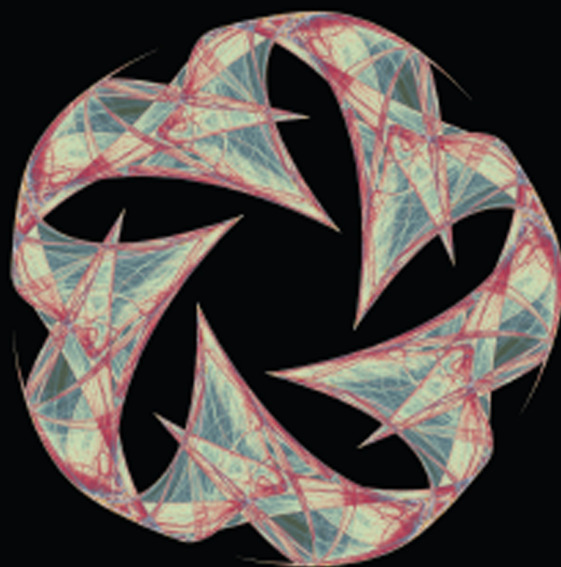
# STUDENT SOLUTIONS MANUAL



A CHAPMAN & HALL BOOK

Mitchal Dichter

## NONLINEAR With Applications to Physics, DYNAMICS Biology, Chemistry, and Engineering AND CHAOS



Steven H. Strogatz

SECOND EDITION

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# Student Solutions Manual for Nonlinear Dynamics and Chaos, Second Edition

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**Mitchal Dichter**



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# CONTENTS

<b>2</b>	<b>Flows on the Line</b>	<b>1</b>
2.1	A Geometric Way of Thinking	1
2.2	Fixed Points and Stability	2
2.3	Population Growth	7
2.4	Linear Stability Analysis	9
2.5	Existence and Uniqueness	11
2.6	Impossibility of Oscillations	13
2.7	Potentials	13
2.8	Solving Equations on the Computer	14
<b>3</b>	<b>Bifurcations</b>	<b>19</b>
3.1	Saddle-Node Bifurcation	19
3.2	Transcritical Bifurcation	27
3.3	Laser Threshold	31
3.4	Pitchfork Bifurcation	33
3.5	Overdamped Bead on a Rotating Hoop	43
3.6	Imperfect Bifurcations and Catastrophes	45
3.7	Insect Outbreak	55
<b>4</b>	<b>Flows on the Circle</b>	<b>65</b>
4.1	Examples and Definitions	65
4.2	Uniform Oscillator	66
4.3	Nonuniform Oscillator	67
4.4	Overdamped Pendulum	75
4.5	Fireflies	77
4.6	Superconducting Josephson Junctions	80
<b>5</b>	<b>Linear Systems</b>	<b>87</b>
5.1	Definitions and Examples	87
5.2	Classification of Linear Systems	92
5.3	Love Affairs	101
<b>6</b>	<b>Phase Plane</b>	<b>103</b>
6.1	Phase Portraits	103
6.2	Existence, Uniqueness, and Topological Consequences	109
6.3	Fixed Points and Linearization	110
6.4	Rabbits versus Sheep	117
6.5	Conservative Systems	129
6.6	Reversible Systems	145
6.7	Pendulum	160
6.8	Index Theory	164

<b>7</b>	<b>Limit Cycles</b>	<b>173</b>
7.1	Examples	173
7.2	Ruling Out Closed Orbits	179
7.3	Poincaré-Bendixson Theorem	188
7.4	Liénard Systems	197
7.5	Relaxation Oscillations	198
7.6	Weakly Nonlinear Oscillators	203
<b>8</b>	<b>Bifurcations Revisited</b>	<b>219</b>
8.1	Saddle-Node, Transcritical, and Pitchfork Bifurcations	219
8.2	Hopf Bifurcations	226
8.3	Oscillating Chemical Reactions	237
8.4	Global Bifurcations of Cycles	241
8.5	Hysteresis in the Driven Pendulum and Josephson Junction	248
8.6	Coupled Oscillators and Quasiperiodicity	253
8.7	Poincaré Maps	267
<b>9</b>	<b>Lorenz Equations</b>	<b>273</b>
9.1	A Chaotic Waterwheel	273
9.2	Simple Properties of the Lorenz Equations	276
9.3	Chaos on a Strange Attractor	279
9.4	Lorenz Map	292
9.5	Exploring Parameter Space	292
9.6	Using Chaos to Send Secret Messages	303
<b>10</b>	<b>One-Dimensional Maps</b>	<b>307</b>
10.1	Fixed Points and Cobwebs	307
10.2	Logistic Map: Numerics	318
10.3	Logistic Map: Analysis	323
10.4	Periodic Windows	331
10.5	Liapunov Exponent	339
10.6	Universality and Experiments	342
10.7	Renormalization	352
<b>11</b>	<b>Fractals</b>	<b>359</b>
11.1	Countable and Uncountable Sets	359
11.2	Cantor Set	360
11.3	Dimension of Self-Similar Fractals	362
11.4	Box Dimension	366
11.5	Pointwise and Correlation Dimensions	369
<b>12</b>	<b>Strange Attractors</b>	<b>371</b>
12.1	The Simplest Examples	371
12.2	Hénon Map	381
12.3	Rössler System	387
12.4	Chemical Chaos and Attractor Reconstruction	389
12.5	Forced Double-Well Oscillator	391

# 2

## Flows on the Line

### 2.1 A Geometric Way of Thinking

---

#### 2.1.1

The fixed points of the flow  $\dot{x} = \sin(x)$  occur when

$$\dot{x} = 0 \Rightarrow \sin(x) = 0 \Rightarrow x = z\pi \quad z \in \mathbb{Z}$$

---

#### 2.1.3

**a)**

We can find the flow's acceleration  $\ddot{x}$  by first deriving an equation containing  $\ddot{x}$  by taking the time derivative of the differential equation.

$$\frac{d}{dt}\dot{x} = \frac{d}{dt}\sin(x) \Rightarrow \ddot{x} = \cos(x)\dot{x}$$

We can obtain  $\ddot{x}$  solely as a function of  $x$  by plugging in our previous equation for  $\dot{x}$ .

$$\ddot{x} = \cos(x)\sin(x)$$

**b)**

We can find what values of  $x$  give the acceleration  $\ddot{x}$  maximum positive values by using the trigonometric identity

$$\frac{1}{2}\sin(2x) = \sin(x)\cos(x)$$

which can be used to rewrite  $\ddot{x}$  as

$$\ddot{x} = \frac{1}{2}\sin(2x)$$

which has maximums when

$$x = \left(z + \frac{1}{4}\right)\pi \quad z \in \mathbb{Z}$$

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#### 2.1.5

**a)**

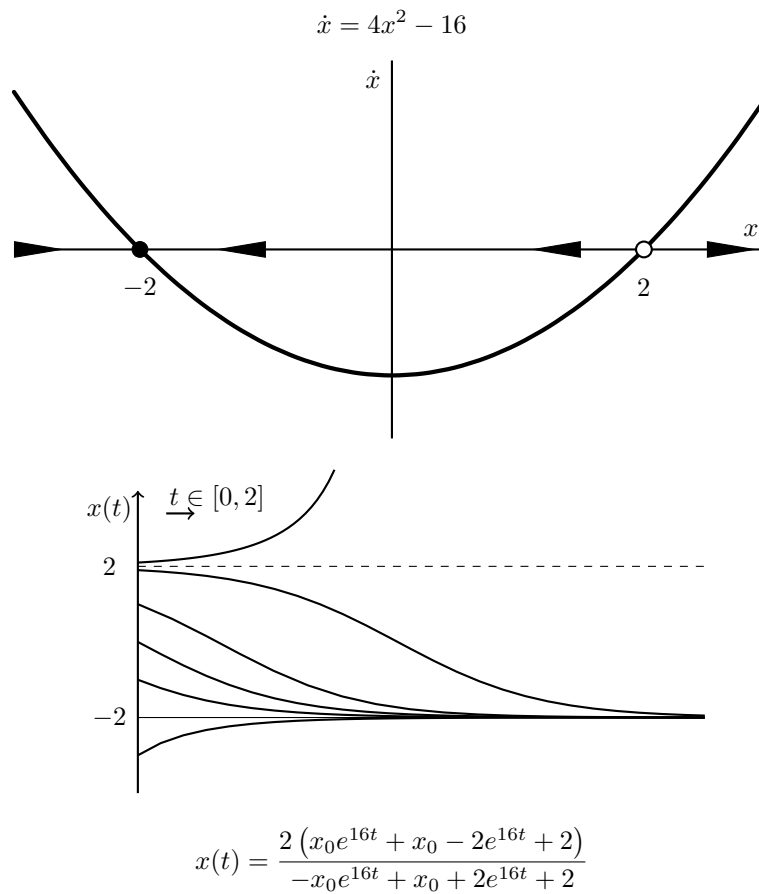
A pendulum submerged in honey with the pendulum at the 12 o'clock position corresponding to  $x = 0$  is qualitatively similar to  $\dot{x} = \sin(x)$ . The force near the 12 o'clock position is small, is greatest at the 3 o'clock position, and is again small at the 6 o'clock position.

**b)**

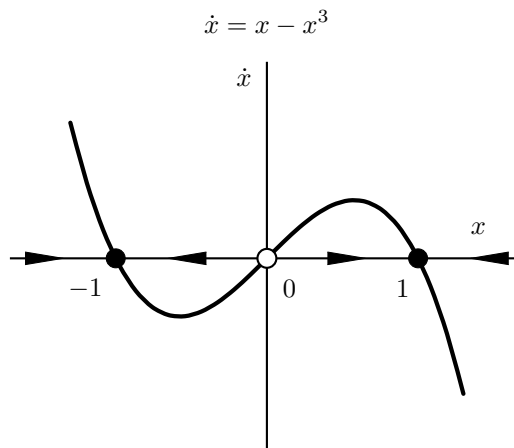
$x = 0$  and  $x = \pi$  being unstable and stable fixed points respectively is consistent with our intuitive understanding of gravity.

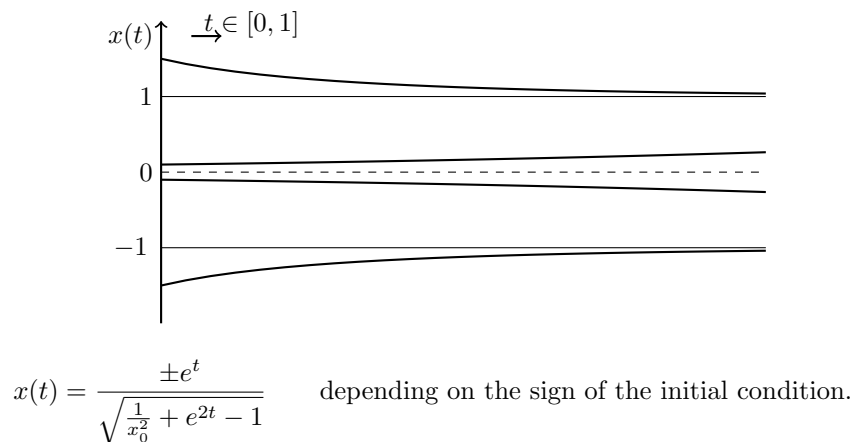
## 2.2 Fixed Points and Stability

### 2.2.1

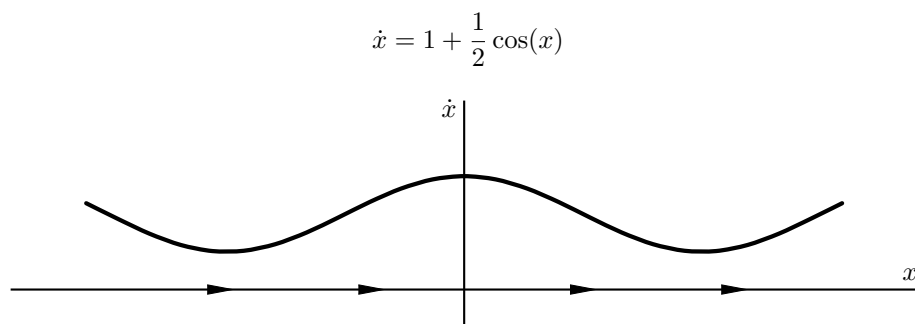


### 2.2.3

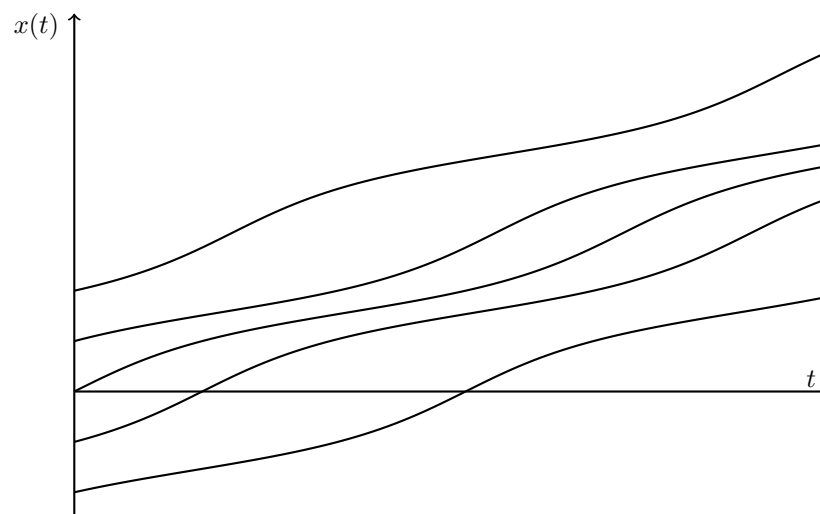




### 2.2.5



There are no fixed points, but the rate increase for  $x(t)$  does vary.



$$x(t) = 2 \arctan \left( \sqrt{3} \tan \left( \arctan \left( \frac{\tan \left( \frac{x_0}{2} \right)}{\sqrt{3}} \right) + \frac{\sqrt{3}t}{4} \right) \right)$$

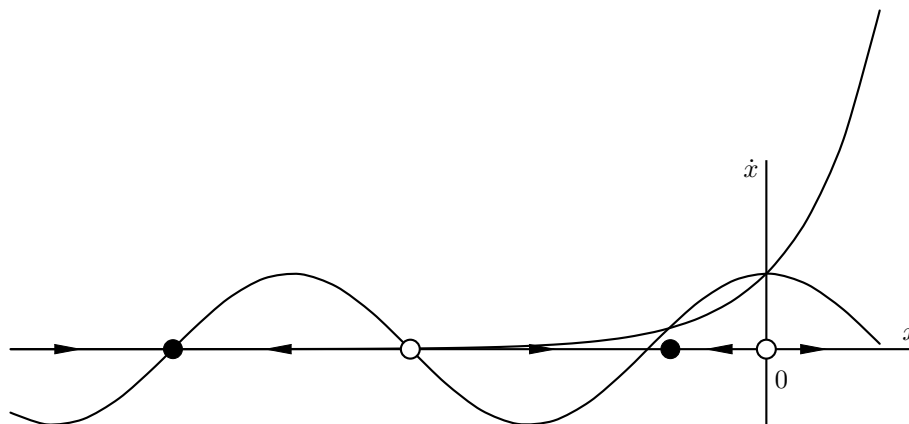


## 4 Chapter 2: Flows on the Line

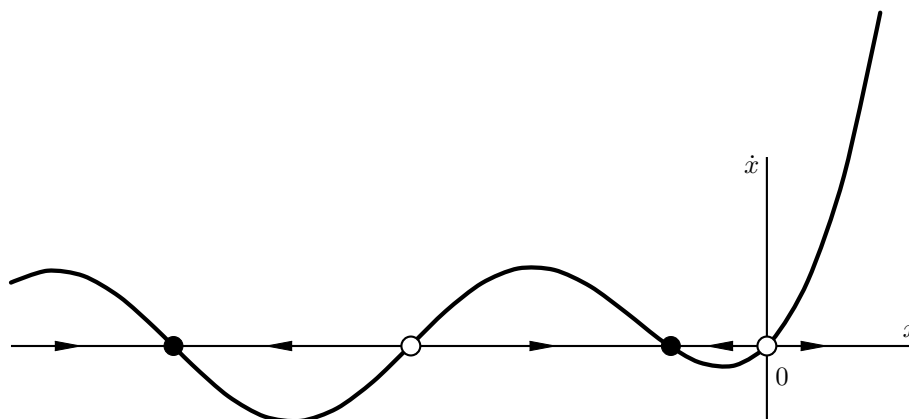
## 2.2.7

$$\dot{x} = e^x - \cos(x)$$

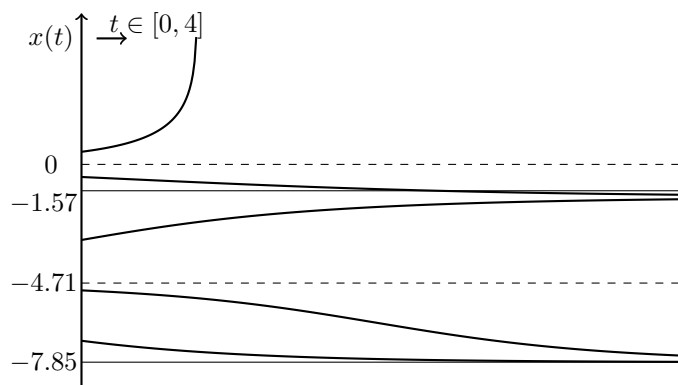
We can't solve for the fixed points analytically, but we can find the fixed points approximately by looking at the intersections of  $e^x$  and  $\cos(x)$ , and determine the stability of the fixed points from which graph is greater than the other nearby.



We could also plot the graph using a computer.

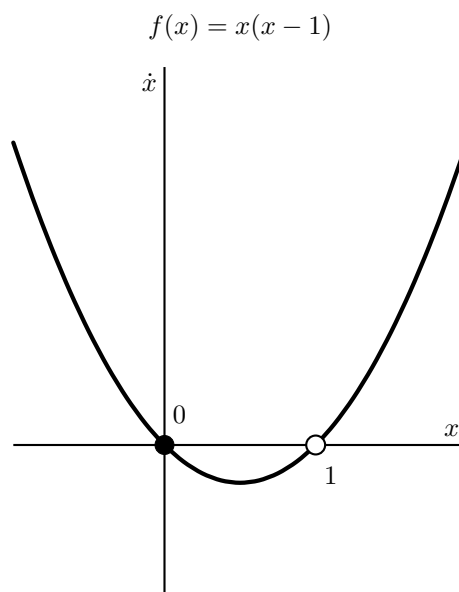


There are fixed points at  $x \approx \pi \left( \frac{1}{2} - n \right)$ ,  $n \in \mathbb{N}$ , and  $x = 0$ .



Unable to find an analytic solution.

### 2.2.9



### 2.2.11

RC circuit

$$\dot{Q} = \frac{V_0}{R} - \frac{Q}{RC} \Rightarrow \dot{Q} + \frac{1}{RC}Q = \frac{V_0}{R} \quad Q(0) = 0$$

Multiply by an integrating factor  $e^{\frac{t}{RC}}$  to both sides.

$$\dot{Q}e^{\frac{t}{RC}} + \frac{1}{RC}e^{\frac{t}{RC}}Q = \frac{V_0}{R}e^{\frac{t}{RC}} \Rightarrow \frac{d}{dt}(Qe^{\frac{t}{RC}}) = \frac{V_0}{R}e^{\frac{t}{RC}}$$