STUDENT SOLUTIONS MANUAL

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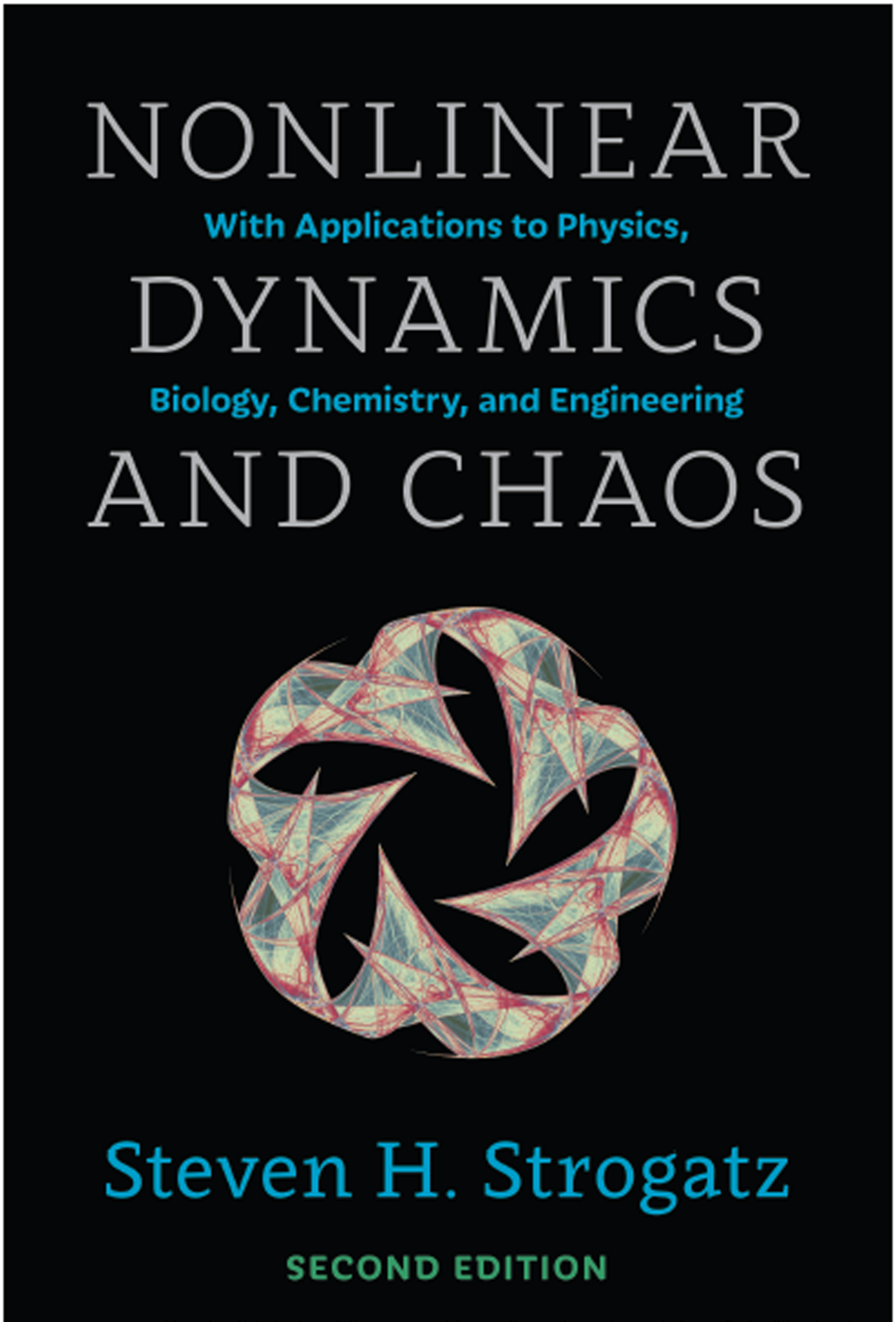
Mitchal Dichter



Student Solutions Manual for Nonlinear Dynamics and Chaos, Second Edition

Mitchal Dichter

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Flows on the Line

2.1 A Geometric Way of Thinking

2.1.1

The fixed points of the flow $\dot{x} = \sin(x)$ occur when

$$\dot{x} = 0 \Rightarrow \sin(x) = 0 \Rightarrow x = z\pi \qquad z \in \mathbb{Z}$$

2.1.3

a)

We can find the flow's acceleration $\ddot{x}$ by first deriving an equation containing $\ddot{x}$ by taking the time derivative of the differential equation.

$$\frac{\mathrm{d}}{\mathrm{d}t}\dot{x} = \frac{\mathrm{d}}{\mathrm{d}t}\sin(x) \Rightarrow \ddot{x} = \cos(x)\dot{x}$$

We can obtain $\ddot{x}$ solely as a function of x by plugging in our previous equation for $\dot{x}$.

$$\ddot{x} = \cos(x)\sin(x)$$

b)

We can find what values of x give the acceleration $\ddot{x}$ maximum positive values by using the trigonometric identity

$$\frac{1}{2}\sin(2x) = \sin(x)\cos(x)$$

which can be used to rewrite $\ddot{x}$ as

$$\ddot{x} = \frac{1}{2}\sin(2x)$$

which has maximums when

$$x = \left(z + \frac{1}{4}\right)\pi \qquad z \in \mathbb{Z}$$

2.1.5

a)

A pendulum submerged in honey with the pendulum at the 12 o'clock position corresponding to $x = 0$ is qualitatively similar to $\dot{x} = \sin(x)$. The force near the 12 o'clock position is small, is greatest at the 3 o'clock position, and is again small at the 6 o'clock position.

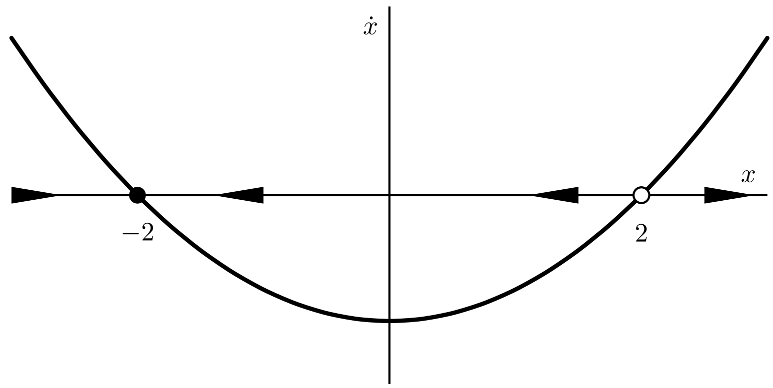
b)

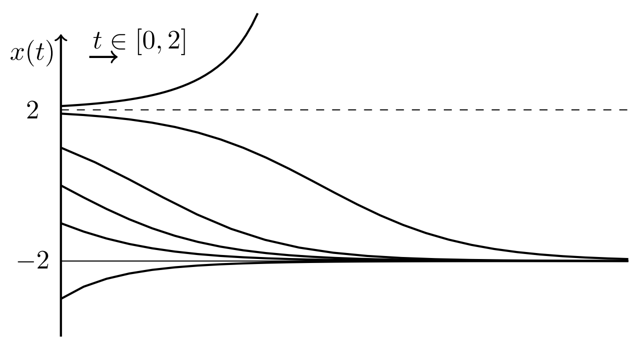
$x = 0$ and $x = \pi$ being unstable and stable fixed points respectively is consistent with our intuitive understanding of gravity.

2.2 Fixed Points and Stability

2.2.1

$$\dot{x} = 4x^2 - 16$$

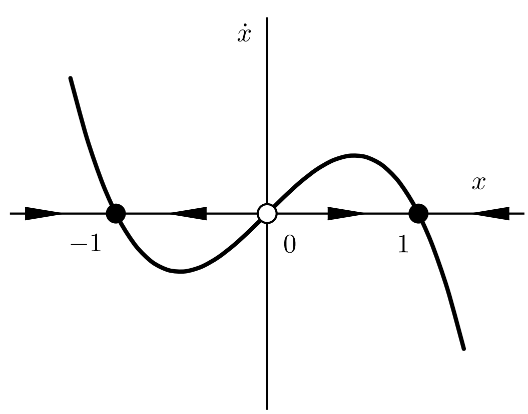


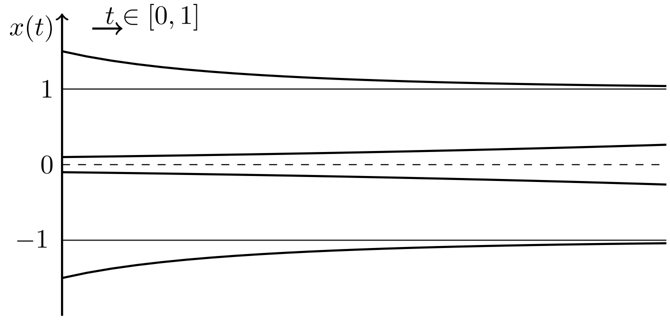


$$x(t) = \frac{2\left(x_0 e^{16t} + x_0 - 2e^{16t} + 2\right)}{-x_0 e^{16t} + x_0 + 2e^{16t} + 2}$$

2.2.3

$$\dot{x} = x - x^3$$

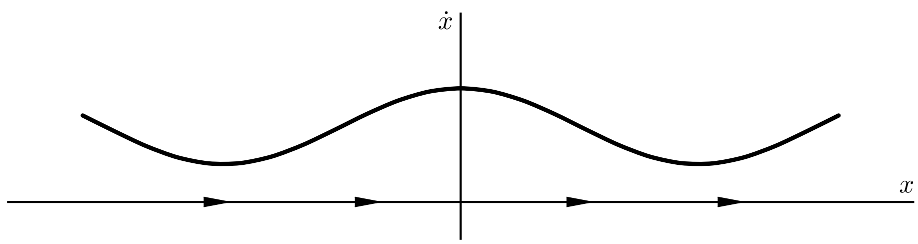




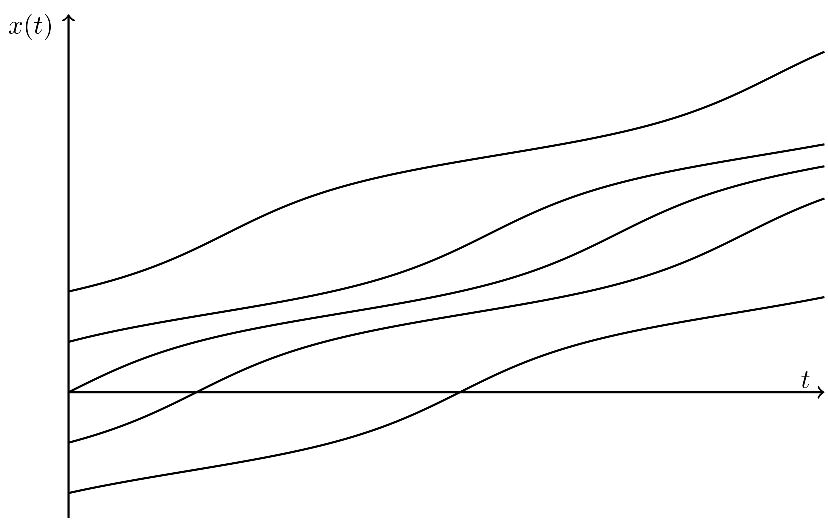
$$x(t) = \frac{\pm e^t}{\sqrt{\frac{1}{x_0^2} + e^{2t} - 1}}$$ depending on the sign of the initial condition.

2.2.5

$$\dot{x} = 1 + \frac{1}{2}\cos(x)$$



There are no fixed points, but the rate increase for $x(t)$ does vary.

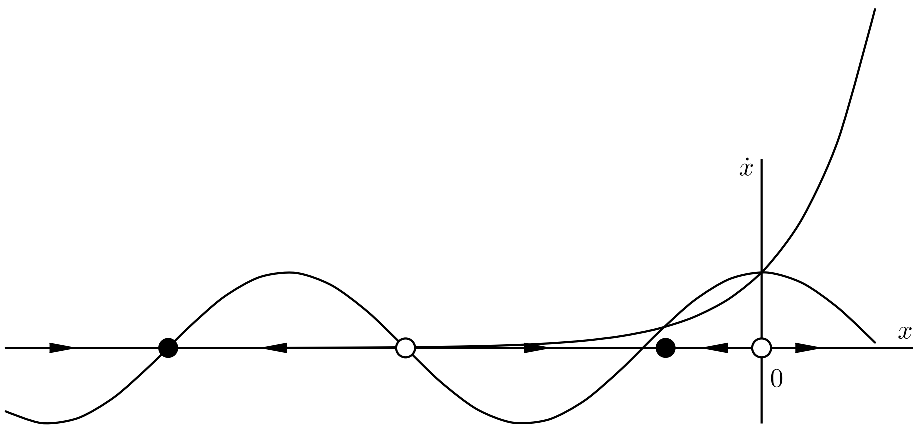


$$x(t) = 2\arctan\left(\sqrt{3}\tan\left(\arctan\left(\frac{\tan\left(\frac{x_0}{2}\right)}{\sqrt{3}}\right) + \frac{\sqrt{3}t}{4}\right)\right)$$

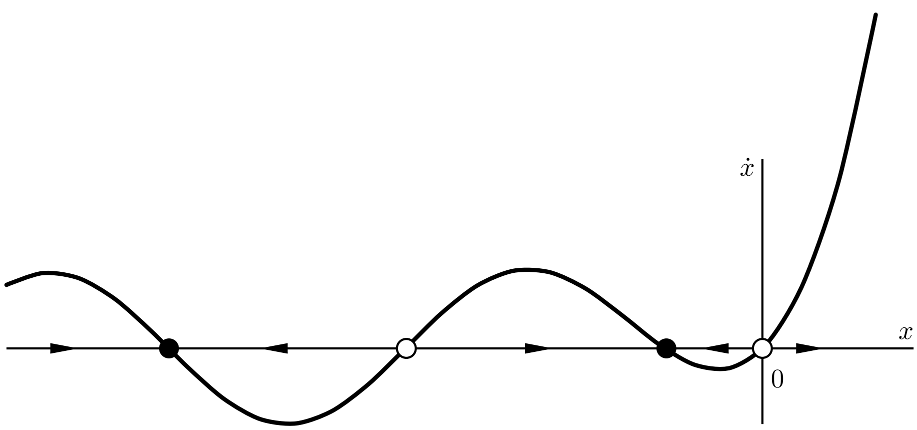
2.2.7

$$\dot{x} = e^x - \cos(x)$$

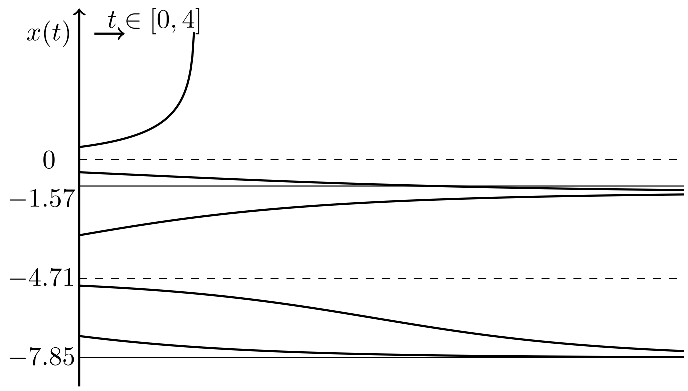
We can't solve for the fixed points analytically, but we can find the fixed points approximately by looking at the intersections of e^x and $\cos(x)$, and determine the stability of the fixed points from which graph is greater than the other nearby.



We could also plot the graph using a computer.

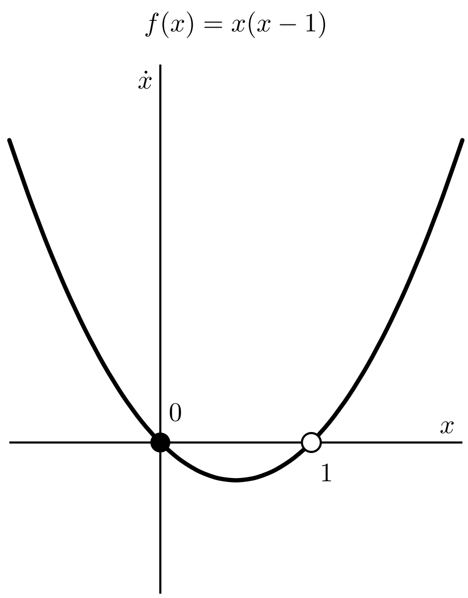


There are fixed points at $x \approx \pi\left(\frac{1}{2} - n\right),\ n \in \mathbb{N}$, and $x = 0$.



Unable to find an analytic solution.

2.2.9



2.2.11

RC circuit

$$\dot{Q} = \frac{V_0}{R} - \frac{Q}{RC} \Rightarrow \dot{Q} + \frac{1}{RC}Q = \frac{V_0}{R} \qquad Q(0) = 0$$

Multiply by an integrating factor $e^{\frac{t}{RC}}$ to both sides.

$$\dot{Q}e^{\frac{t}{RC}} + \frac{1}{RC}e^{\frac{t}{RC}}Q = \frac{V_0}{R}e^{\frac{t}{RC}} \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(Qe^{\frac{t}{RC}}\right) = \frac{V_0}{R}e^{\frac{t}{RC}}$$