

STUDENT SOLUTIONS MANUAL



A CHAPMAN & HALL BOOK

Mitchal Dichter

NONLINEAR DYNAMICS AND CHAOS

With Applications to Physics,
Biology, Chemistry, and Engineering



Steven H. Strogatz

SECOND EDITION

Student Solutions Manual for Nonlinear Dynamics and Chaos, Second Edition

Mitchal Dichter



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2

Flows on the Line

2.1 A Geometric Way of Thinking

2.1.1

The fixed points of the flow $\dot{x} = \sin(x)$ occur when

$$\dot{x} = 0 \Rightarrow \sin(x) = 0 \Rightarrow x = z\pi \quad z \in \mathbb{Z}$$

2.1.3

a)

We can find the flow's acceleration \ddot{x} by first deriving an equation containing \ddot{x} by taking the time derivative of the differential equation.

$$\frac{d}{dt}\dot{x} = \frac{d}{dt}\sin(x) \Rightarrow \ddot{x} = \cos(x)\dot{x}$$

We can obtain \ddot{x} solely as a function of x by plugging in our previous equation for \dot{x} .

$$\ddot{x} = \cos(x)\sin(x)$$

b)

We can find what values of x give the acceleration \ddot{x} maximum positive values by using the trigonometric identity

$$\frac{1}{2}\sin(2x) = \sin(x)\cos(x)$$

which can be used to rewrite \ddot{x} as

$$\ddot{x} = \frac{1}{2}\sin(2x)$$

which has maximums when

$$x = \left(z + \frac{1}{4}\right)\pi \quad z \in \mathbb{Z}$$

2.1.5

a)

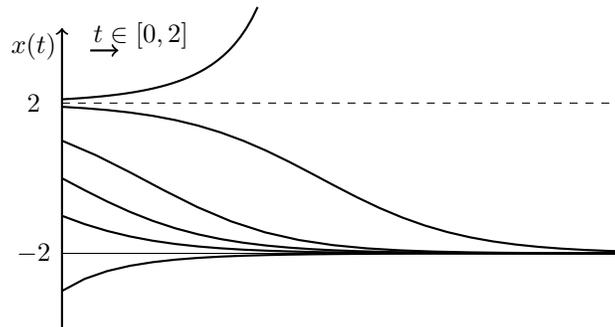
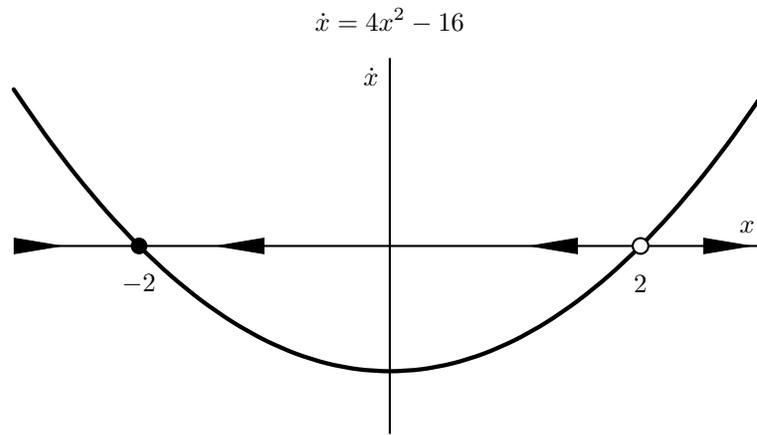
A pendulum submerged in honey with the pendulum at the 12 o'clock position corresponding to $x = 0$ is qualitatively similar to $\dot{x} = \sin(x)$. The force near the 12 o'clock position is small, is greatest at the 3 o'clock position, and is again small at the 6 o'clock position.

b)

$x = 0$ and $x = \pi$ being unstable and stable fixed points respectively is consistent with our intuitive understanding of gravity.

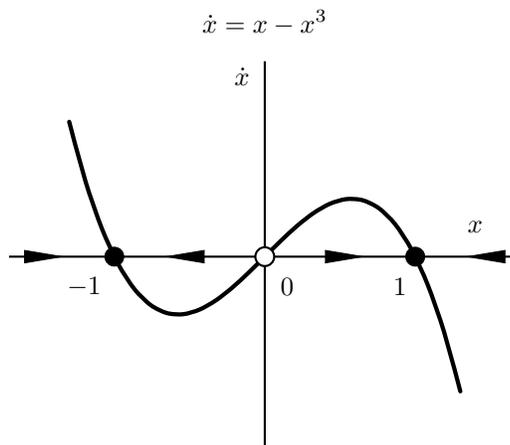
2.2 Fixed Points and Stability

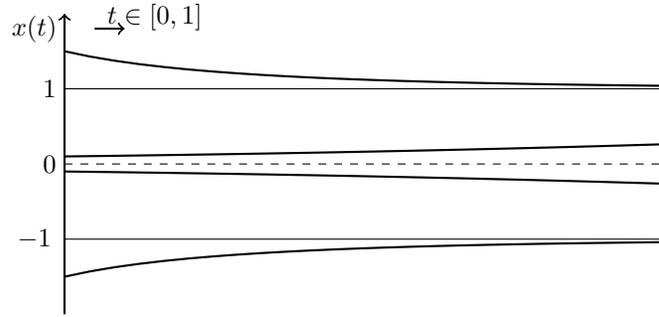
2.2.1



$$x(t) = \frac{2(x_0 e^{16t} + x_0 - 2e^{16t} + 2)}{-x_0 e^{16t} + x_0 + 2e^{16t} + 2}$$

2.2.3

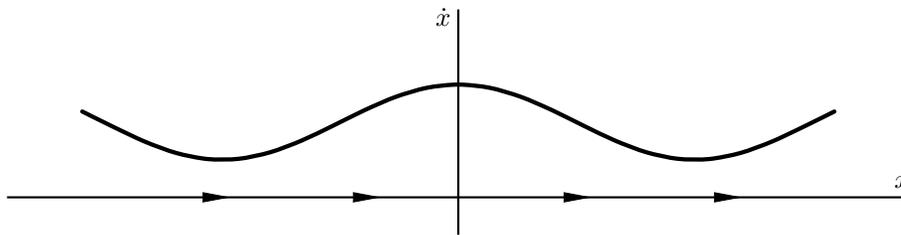




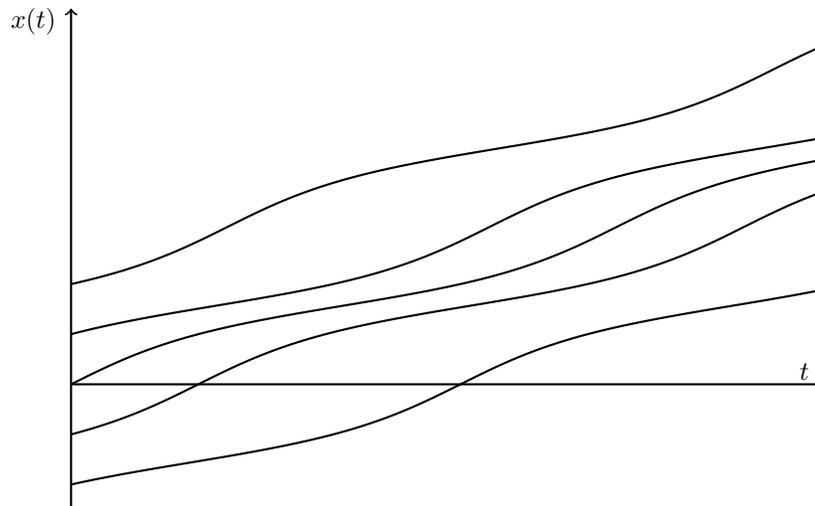
$$x(t) = \frac{\pm e^t}{\sqrt{\frac{1}{x_0^2} + e^{2t} - 1}} \quad \text{depending on the sign of the initial condition.}$$

2.2.5

$$\dot{x} = 1 + \frac{1}{2} \cos(x)$$



There are no fixed points, but the rate increase for $x(t)$ does vary.



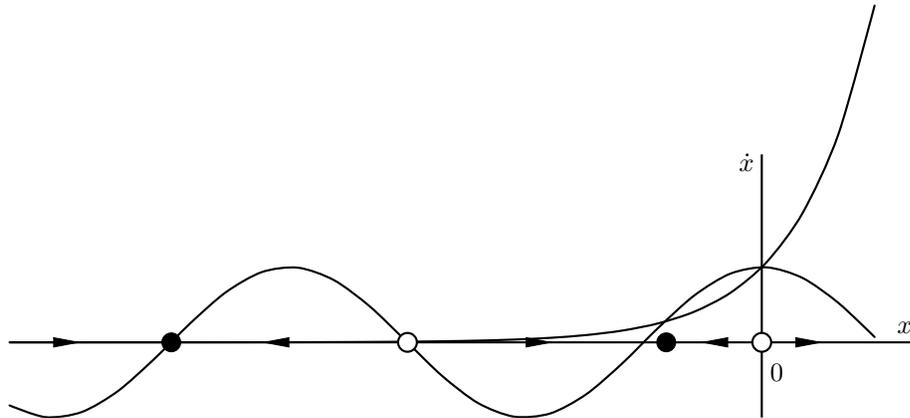
$$x(t) = 2 \arctan \left(\sqrt{3} \tan \left(\arctan \left(\frac{\tan(\frac{x_0}{2})}{\sqrt{3}} \right) + \frac{\sqrt{3}t}{4} \right) \right)$$

4 Chapter 2: Flows on the Line

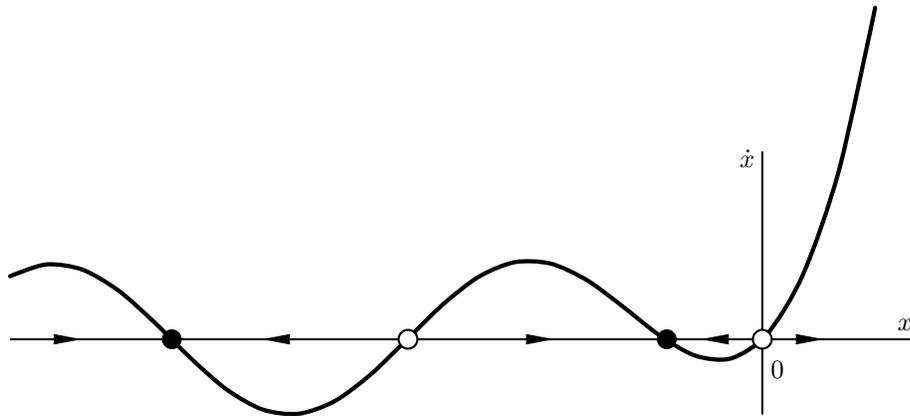
2.2.7

$$\dot{x} = e^x - \cos(x)$$

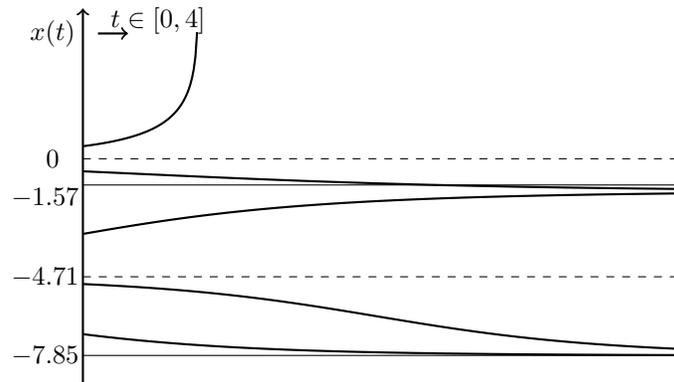
We can't solve for the fixed points analytically, but we can find the fixed points approximately by looking at the intersections of e^x and $\cos(x)$, and determine the stability of the fixed points from which graph is greater than the other nearby.



We could also plot the graph using a computer.

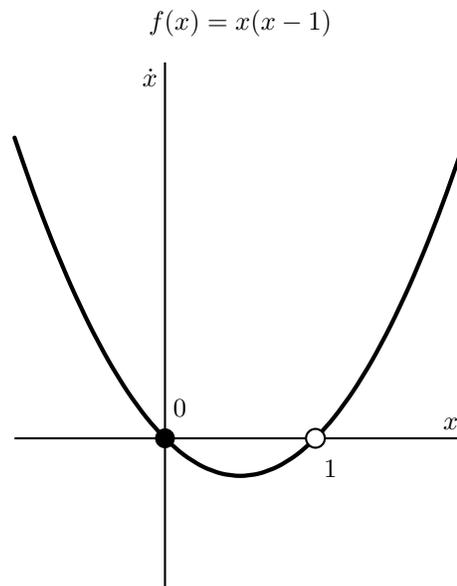


There are fixed points at $x \approx \pi \left(\frac{1}{2} - n \right)$, $n \in \mathbb{N}$, and $x = 0$.



Unable to find an analytic solution.

2.2.9



2.2.11

RC circuit

$$\dot{Q} = \frac{V_0}{R} - \frac{Q}{RC} \Rightarrow \dot{Q} + \frac{1}{RC}Q = \frac{V_0}{R} \quad Q(0) = 0$$

Multiply by an integrating factor $e^{\frac{t}{RC}}$ to both sides.

$$\dot{Q}e^{\frac{t}{RC}} + \frac{1}{RC}e^{\frac{t}{RC}}Q = \frac{V_0}{R}e^{\frac{t}{RC}} \Rightarrow \frac{d}{dt} \left(Qe^{\frac{t}{RC}} \right) = \frac{V_0}{R}e^{\frac{t}{RC}}$$