## Linear Algebra

and its applications

FOURTH EDITION



INSTRUCTOR SOLUTIONS MANUAL

David C. Lay

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## INSTRUCTOR'S SOLUTIONS MANUAL

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# LINEAR ALGEBRA AND ITS APPLICATIONS FOURTH EDITION

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### Linear Equations in Linear Algebra

#### 1.1 SOLUTIONS

**Notes**: The key exercises are 7 (or 11 or 12), 19–22, and 25. For brevity, the symbols R1, R2,..., stand for row 1 (or equation 1), row 2 (or equation 2), and so on. Additional notes are at the end of the section.

1. 
$$x_1 + 5x_2 = 7$$
  $\begin{bmatrix} 1 & 5 & 7 \\ -2x_1 - 7x_2 = -5 \end{bmatrix}$ 

Replace R2 by 
$$R2 + (2)R1$$
 and obtain:

Scale R2 by 1/3: 
$$x_1 + 5x_2 = 7 \\ x_2 = 3$$
 
$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$
 Replace R1 by R1 + (-5)R2: 
$$x_1 = -8 \\ x_2 = 3$$
 
$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$$

The solution is  $(x_1, x_2) = (-8, 3)$ , or simply (-8, 3).

2. 
$$3x_1 + 6x_2 = -3$$
  $\begin{bmatrix} 3 & 6 & -3 \\ 5x_1 + 7x_2 = 10 & 5 & 7 & 10 \end{bmatrix}$ 

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Scale R1 by 1/3 and obtain: 
$$x_1 + 2x_2 = -1 \\ 5x_1 + 7x_2 = 10$$
 
$$\begin{bmatrix} 1 & 2 & -1 \\ 5 & 7 & 10 \end{bmatrix}$$

Replace R2 by R2 + (-5)R1: 
$$x_1 + 2x_2 = -1$$

$$-3x_1 = 15$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 15 \end{bmatrix}$$

Replace R2 by R2 + (-5)R1: 
$$x_1 + 2x_2 = -1 \\ -3x_2 = 15$$
 
$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 15 \end{bmatrix}$$
 Scale R2 by -1/3: 
$$x_1 + 2x_2 = -1 \\ x_2 = -5$$
 
$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -5 \end{bmatrix}$$

Replace R1 by R1 + (-2)R2: 
$$x_1 = 9 \\ x_2 = -5$$
 
$$\begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -5 \end{bmatrix}$$

The solution is  $(x_1, x_2) = (9, -5)$ , or simply (9, -5).

**3**. The point of intersection satisfies the system of two linear equations:

$$x_1 + 2x_2 = 4$$
  $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ 

The point of intersection is  $(x_1, x_2) = (2, 1)$ .

**4**. The point of intersection satisfies the system of two linear equations:

The point of intersection is  $(x_1, x_2) = (-3, -5)$ .

5. The system is already in "triangular" form. The fourth equation is  $x_4 = -5$ , and the other equations do not contain the variable  $x_4$ . The next two steps should be to use the variable  $x_3$  in the third equation to eliminate that variable from the first two equations. In matrix notation, that means to replace R2 by its sum with -4 times R3, and then replace R1 by its sum with 3 times R3.

6. One more step will put the system in triangular form. Replace R4 by its sum with -4 times R3, which

produces 
$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -7 & 14 \end{bmatrix}$$
. After that, the next step is to scale the fourth row by  $-1/7$ .

7. Ordinarily, the next step would be to interchange R3 and R4, to put a 1 in the third row and third column. But in this case, the third row of the augmented matrix corresponds to the equation  $0 x_1 + 0 x_2 + 0 x_3 = 1$ , or simply, 0 = 1. A system containing this condition has no solution. Further row operations are unnecessary once an equation such as 0 = 1 is evident. The solution set is empty.

**8**. The standard row operations are:

$$\begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The solution set contains one solution: (0, 0, 0, 0).

9. The system has already been reduced to triangular form. Begin by replacing R3 by R3 + (3)R4:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & 0 & 14 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Next, replace R2 by R2 + (2)R3. Finally, replace R1 by R1 + R2:

$$\begin{bmatrix}
1 & -1 & 0 & 0 & -5 \\
0 & 1 & 0 & 0 & 21 \\
0 & 0 & 1 & 0 & 14 \\
0 & 0 & 0 & 1 & 4
\end{bmatrix} \sim \begin{bmatrix}
1 & 0 & 0 & 0 & 16 \\
0 & 1 & 0 & 0 & 21 \\
0 & 0 & 1 & 0 & 14 \\
0 & 0 & 0 & 1 & 4
\end{bmatrix}$$

The solution set contains one solution: (16, 21, 14, 4).

10. The system has already been reduced to triangular form. Use the 1 in the fourth row to change the 3 and -2 above it to zeros. That is, replace R2 by R2 + (-3)R4 and replace R1 by R1 + (2)R4. For the final step, replace R1 by R1 + (-3)R2.

$$\begin{bmatrix} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 0 & -11 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -47 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

The solution set contains one solution: (-47, 12, 2, -2).

11. First, swap R1 and R2. Then replace R3 by R3 + (-2)R1. Finally, replace R3 by R3 + (1)R2.

$$\begin{bmatrix} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

The system is inconsistent, because the last row would require that 0 = -2 if there were a solution. The solution set is empty.

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- 12. Replace R2 by R2 + (-2)R1 and replace R3 by R3 + (2)R1. Finally, replace R3 by R3 + (3)R2.

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

The system is inconsistent, because the last row would require that 0 = 5 if there were a solution. The solution set is empty.

13.  $\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$ 

$$\sim \begin{bmatrix}
 1 & 0 & -3 & 8 \\
 0 & 1 & 5 & -2 \\
 0 & 0 & 1 & -1
\end{bmatrix} \sim \begin{bmatrix}
 1 & 0 & 0 & 5 \\
 0 & 1 & 0 & 3 \\
 0 & 0 & 1 & -1
\end{bmatrix}$$
. The solution is (5, 3, -1).

14.  $\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{bmatrix}$ 

$$\begin{bmatrix}
1 & 0 & -3 & -4 \\
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2
\end{bmatrix} \sim \begin{bmatrix}
1 & 0 & -3 & -4 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 2
\end{bmatrix} \sim \begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 2
\end{bmatrix}.$$
 The solution is  $(2, -1, 2)$ .

**15**. First, replace R3 by R3 + (1)R1, then replace R4 by R4 + (1)R2, and finally replace R4 by R4 + (–1)R3.

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & -1 & 5 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & -8 \end{bmatrix}$$

The system is inconsistent, because the last row would require that 0 = -8 if there were a solution.

**16**. First replace R4 by R4 + (3/2)R1 and replace R4 by R4 + (-2/3)R2. (One could also scale R1 and R2 before adding to R4, but the arithmetic is rather easy keeping R1 and R2 unchanged.) Finally, replace R4 by R4 + (-1)R3.

$$\begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & -5 & -10 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & -5 & -10 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & -9 & -9 \end{bmatrix}$$

The system is now in triangular form and has a solution. In fact, using the argument from Example 2, one can see that the solution is unique.

17. Row reduce the augmented matrix corresponding to the given system of three equations:

$$\begin{bmatrix} 2 & 3 & -1 \\ 6 & 5 & 0 \\ 2 & -5 & 7 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & -1 \\ 0 & -4 & 3 \\ 0 & -8 & 8 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & -1 \\ 0 & -4 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

The third equation, 0 = 2, shows that the system is inconsistent, so the three lines have no point in common.

18. Row reduce the augmented matrix corresponding to the given system of three equations:

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{bmatrix}$$

The system is consistent, and using the argument from Example 2, there is only one solution. So the three planes have only one point in common.

- **19.**  $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & h & 4 \\ 0 & 6 3h & -4 \end{bmatrix}$  Write *c* for 6 3h. If c = 0, that is, if h = 2, then the system has no solution, because 0 cannot equal -4. Otherwise, when  $h \neq 2$ , the system has a solution.
- **20.**  $\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & h & -5 \\ 0 & -8 2h & 16 \end{bmatrix}$  Write c for -8 2h. If c = 0, that is, if h = -4, then the system has no solution, because 0 cannot equal 16. Otherwise, when  $h \neq -4$ , the system has a solution.
- **21.**  $\begin{bmatrix} 1 & 4 & -2 \\ 3 & h & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -2 \\ 0 & h-12 & 0 \end{bmatrix}$  Write c for h-12. Then the second equation  $cx_2 = 0$  has a solution for every value of c. So the system is consistent for all h.
- 22.  $\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix} \sim \begin{bmatrix} -4 & 12 & h \\ 0 & 0 & -3 + \frac{h}{2} \end{bmatrix}$  The system is consistent if and only if  $-3 + \frac{h}{2} = 0$ , that is, if and only if h = 6.
- 23. a. True. See the remarks following the box titled *Elementary Row Operations*.
  - **b**. False. A  $5 \times 6$  matrix has five rows.
  - **c**. False. The description applies to a single solution. The solution *set* consists of all possible solutions. Only in special cases does the solution set consist of exactly one solution. Mark a statement True only if the statement is *always* true.
  - **d**. True. See the box before Example 2.
- **24**. **a**. False. The definition of *row equivalent* requires that there exist a sequence of row operations that transforms one matrix into the other.
  - **b.** True. See the box preceding the subsection titled *Existence and Uniqueness Questions*.
  - **c.** False. The definition of *equivalent systems* is in the second paragraph after equation (2).
  - **d**. True. By definition, a consistent system has at least one solution.